Module 3

Light and Wave optics

3.1. The Nature of Light and the Principles of Ray Optics

Dual Nature of light

- Light is light.
- Light exhibits the characteristics of a wave in some situations (reflection, refraction, diffraction, interference, polarization).
- Light exhibits the characteristics of a particle in other situations (photoelectric effect (chapter 40), also reflection & refraction). Light particles are called photons and have energy:

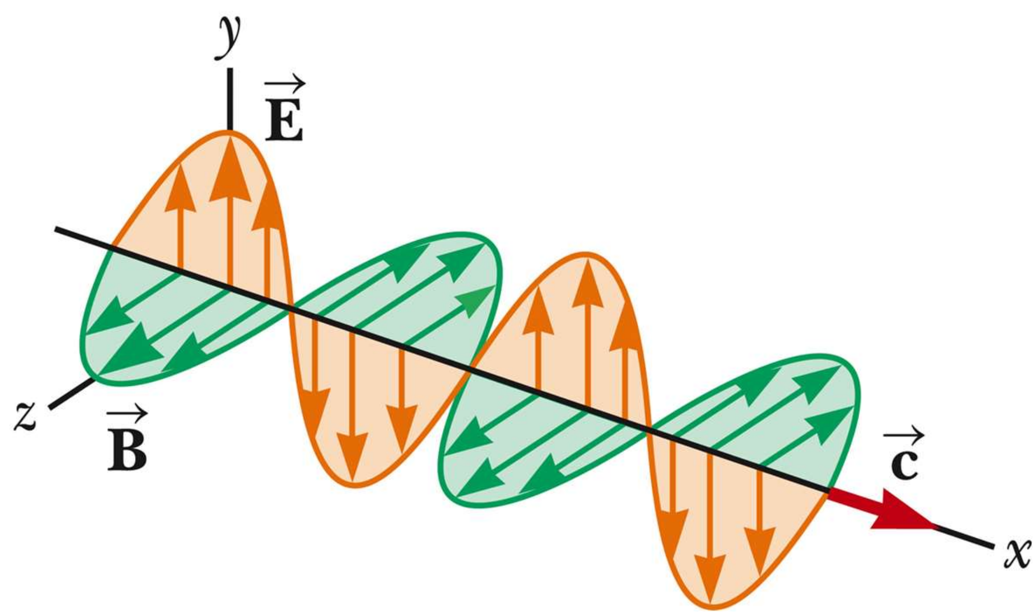
$$E = hf$$

f is frequency of light (wave!), and $h = 6.63 \cdot 10^{-34}$ J·s (Planck's constant)

Light as an electromagnetic wave

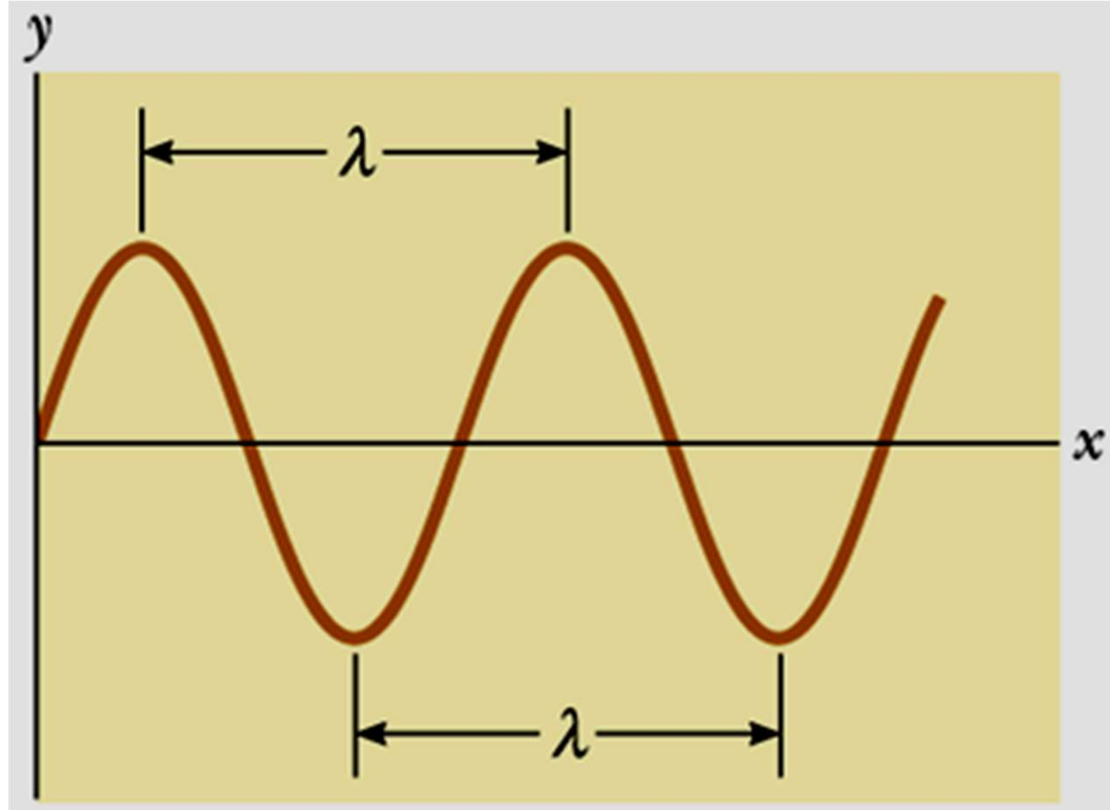
- Light is a transverse electromagnetic wave in which the E-field, B-field and propagation direction are all perpendicular to each other.
- Light sources emit a large number of waves having some particular orientation of the E-field vector.
- Unpolarized light: E-field vectors all point in a random direction.

Schematic depiction of light wave:



Basic Variables of Wave Motion

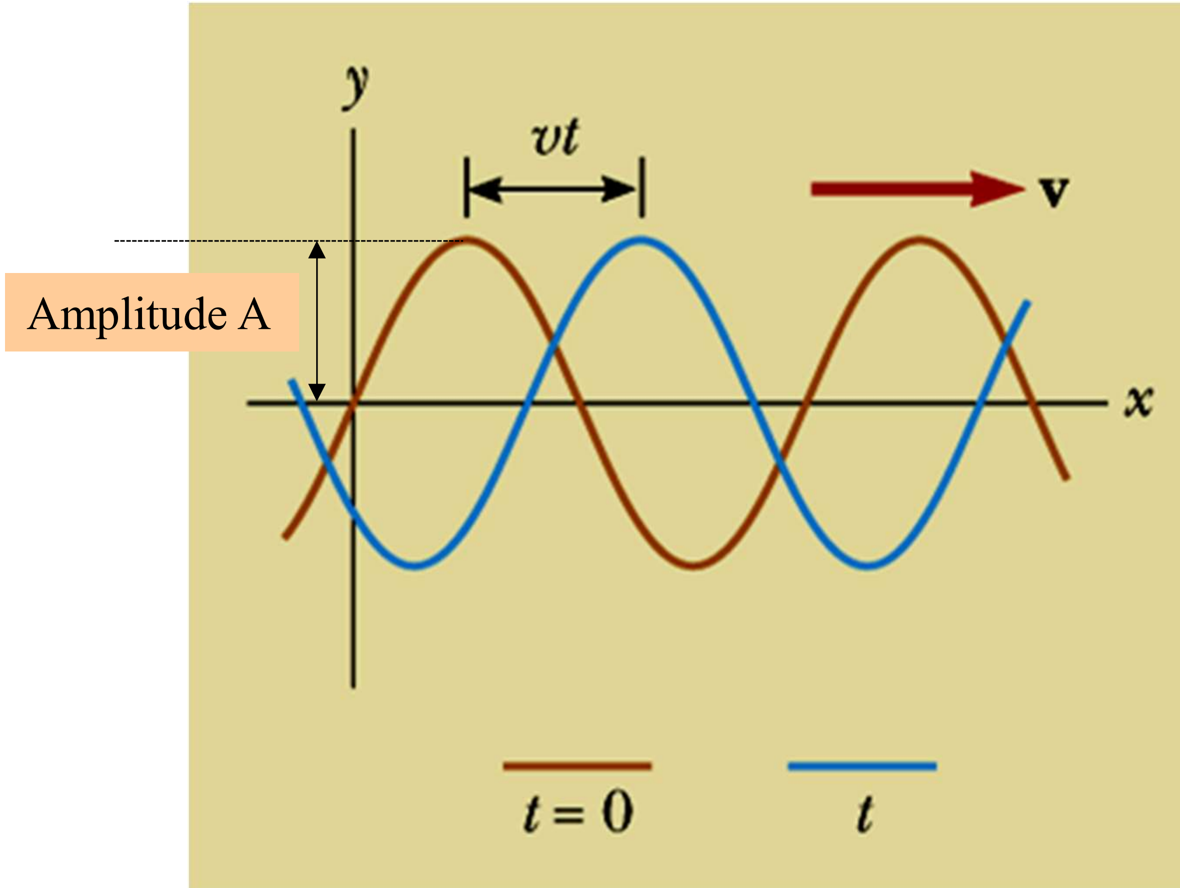
Terminology to describe waves



Snapshot of sinusoidal wave at one time point

- Wavelength λ: Distance from one crest to the next crest.
- Wavelength λ: Distance between two identical points on a wave.
- Period T: Time between the arrival of two adjacent waves (crests).
- Frequency f: 1/T, number of crest that pass a given point per unit time

Traveling sinusoidal wave



Here:
Propagation direction is along x, Amplitude (e.g. E-field) is along y.

$$y = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

$$y = A\sin(kx - \omega t)$$

$$\text{Velocity of wave}: \; v = \frac{\lambda}{T} = f \cdot \lambda$$

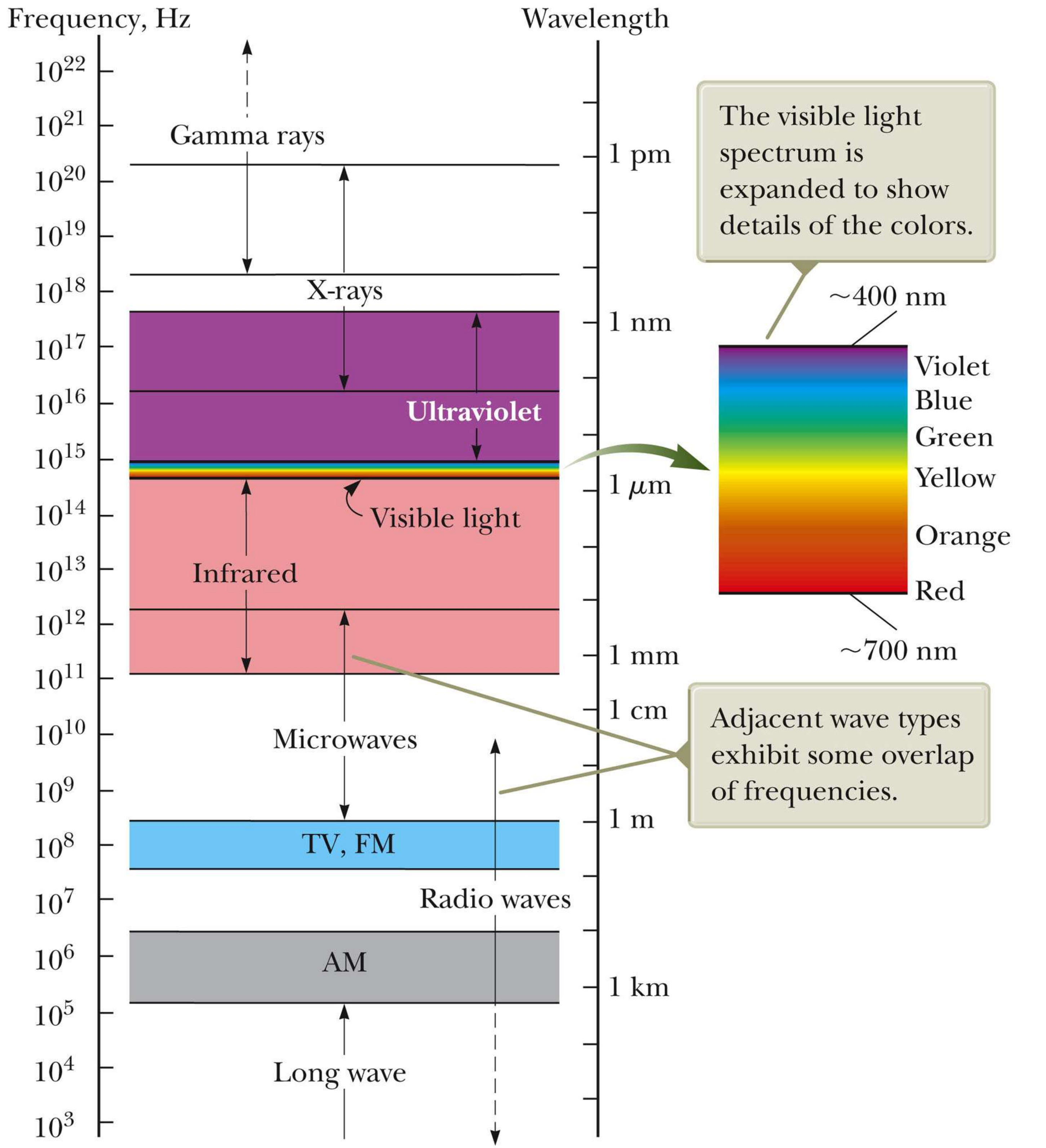
$$\text{angular wave number}: \; k \equiv \frac{2\pi}{\lambda}$$

$$\text{angular frequency}: \; \omega \equiv \frac{2\pi}{T} = 2\pi \cdot f$$

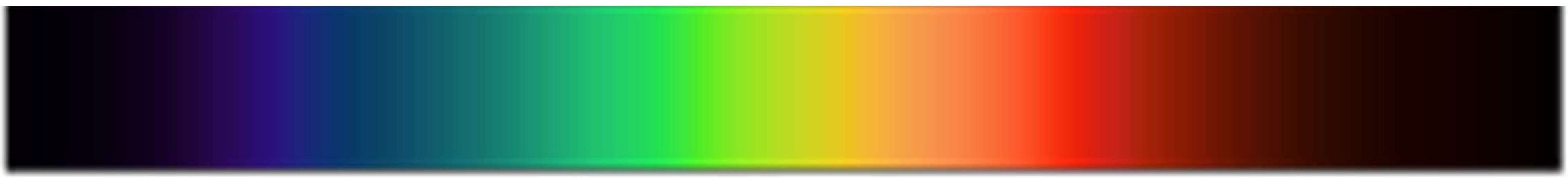
For visible light (see next slide):

- Wavelength, λ, ranges from 380 nm (violet) to 750 nm (red)
- Velocity is speed of light: $3.00 \cdot 10^{8}$ m/s (in vacuum)
- Frequency, f, ranges from $400 \cdot 10^{8}$ Hz (red) to $790 \cdot 10^{8}$ Hz (violet).

The electro-magnetic spectrum



Wavelength, frequency and energy of visible light

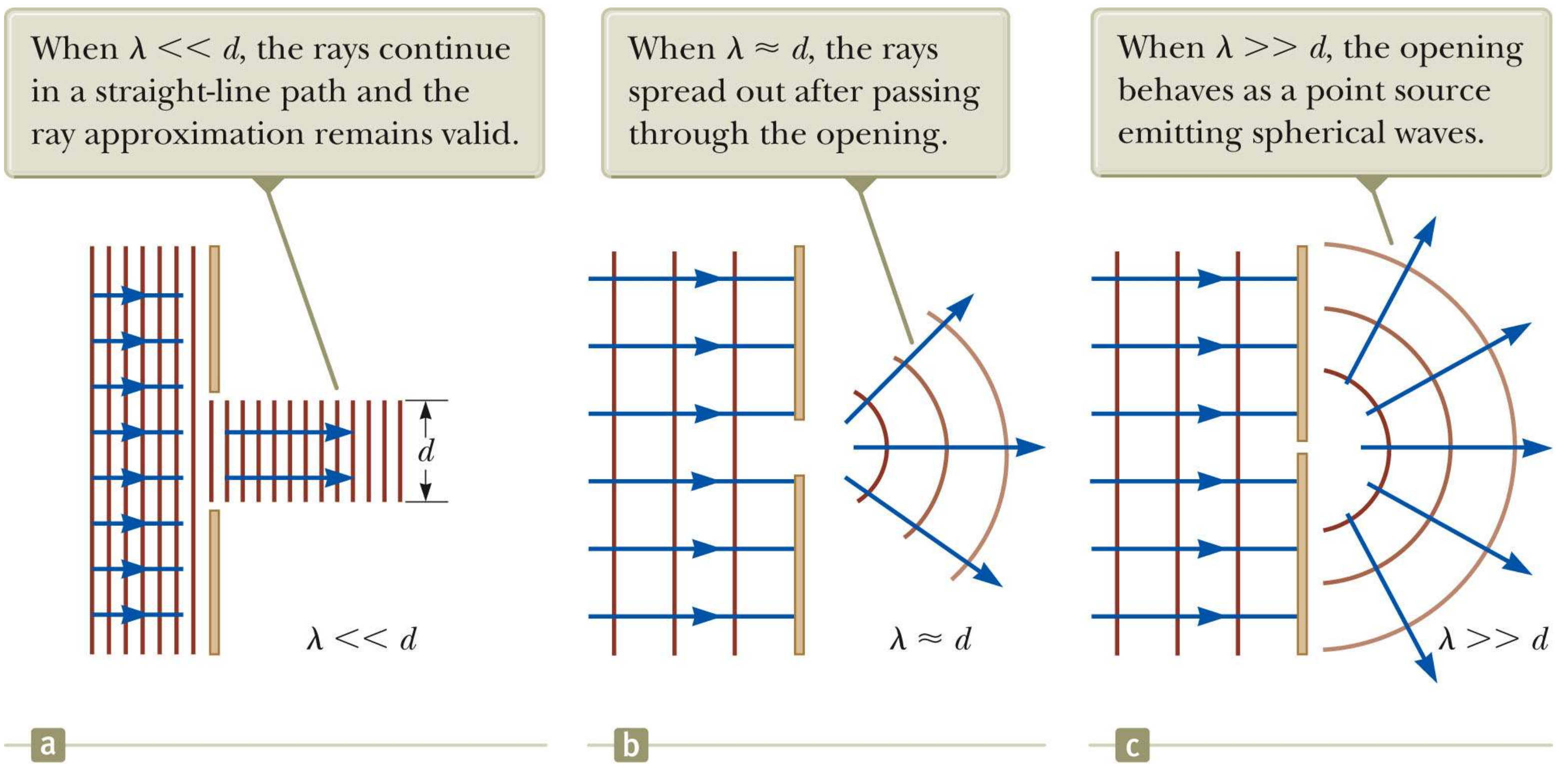


Color	Wavelength	Frequency	Photon energy, $E = hf$
violet	380–450 nm	668–789 THz	2.75–3.26 eV
blue	450–495 nm	606–668 THz	2.50–2.75 eV
green	495–570 nm	526–606 THz	2.17–2.50 eV
yellow	570–590 nm	508–526 THz	2.10–2.17 eV
orange	590–620 nm	484–508 THz	2.00–2.10 eV
red	620–750 nm	400–484 THz	1.65–2.00 eV

From: http://en.wikipedia.org/wiki/Visible_spectrum

The Ray Approximation (geometric optics)

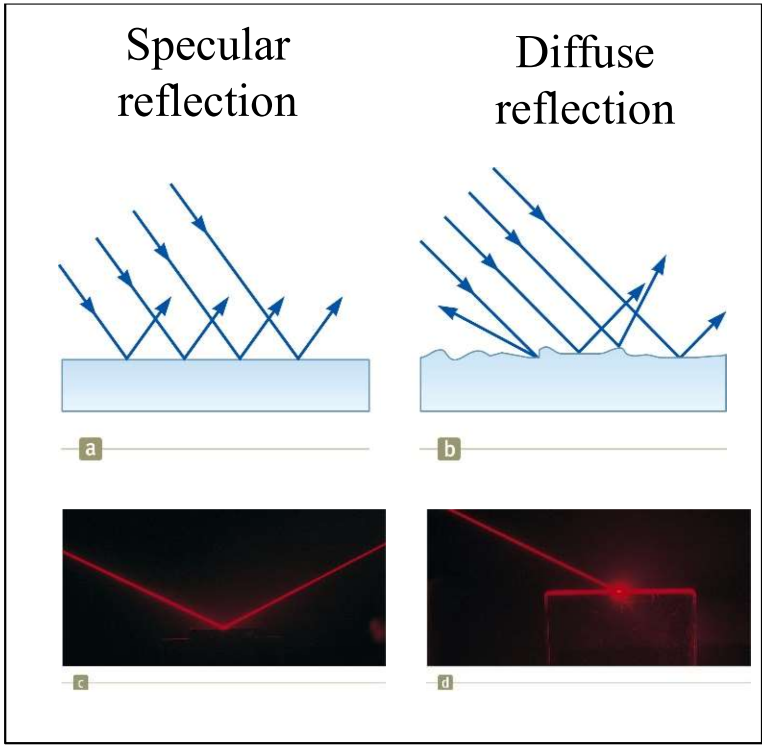
- **Ray approximation:** The rays of a given wave are straight lines along the propagation direction, perpendicular to the wave front.
- Good approximation when features are a larger than the wavelength of light; $\lambda \ll d$.
- When we see an object, light rays coming from the object enter our eye.

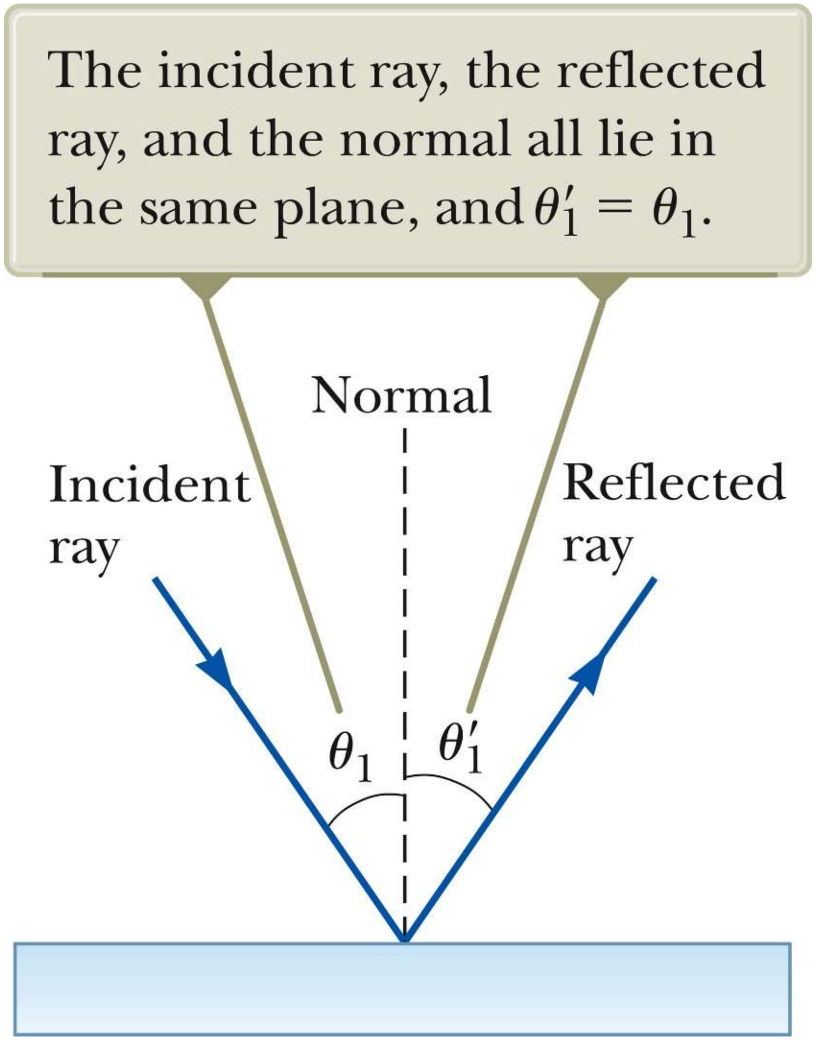


When $\lambda \gg d$, we get phenomena like diffraction and interference

The law of reflection

Incident angle = reflected angle





$$\theta_1 = \theta_1'$$

Incident angle = reflected angle

Angle is measured with respect to normal

Subscript 1 means we are staying in the same medium

The Speed of Light in Materials

- The speed of light <u>in vacuum</u>, c, is the same for all wavelengths of light, no matter the source or other nature of light:

$$Speed\ of\ light\ in\ vacuum: c = 3.00 \cdot 10^8\ m/s$$

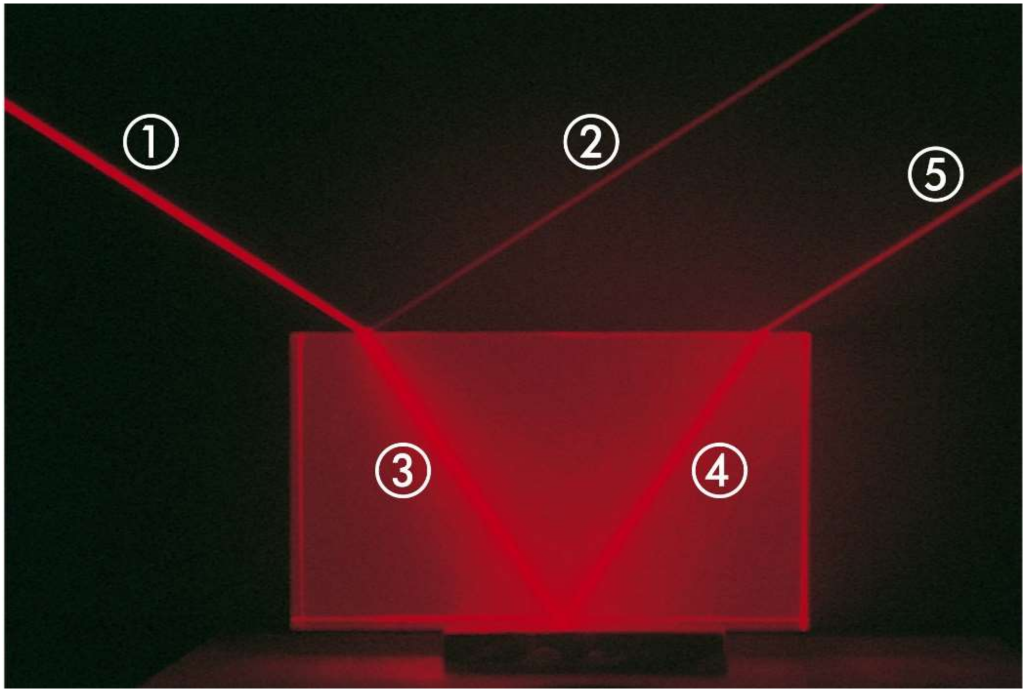
- Inside materials, however, the speed of light can be different
- Materials contain atoms, made of nuclei and electrons
- The electric field from EM waves push on the electrons
- The electrons must move in response
- This generally slows the wave down
- $Speed\ of\ light\ in\ materials:\ v = \frac{c}{n}$
- n is called the index of refraction

Indices of Refraction

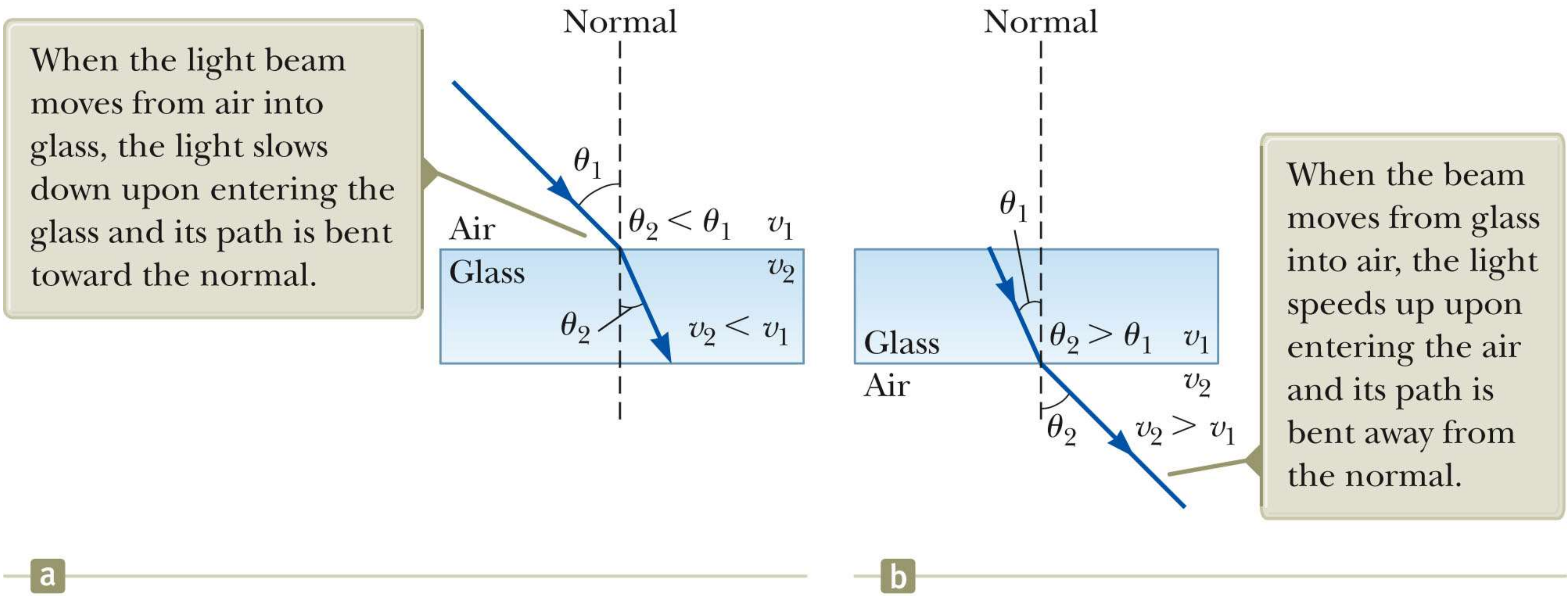
Air (STP)	1.0003
Water	1.333
Ethyl alcohol	1.361
Glycerin	1.473
Fused Quartz	1.434
Glass	1.5 -ish
Cubic zirconia	2.20
Diamond	2.419

Refraction and Snell's law

- When a ray of light travels through a transparent medium, part of the light is reflected and part of the light enters the second medium.
- The ray that enters the second medium changes direction, **it is refracted.**
- The incident ray, the reflected ray and the refracted ray all lie in the same plane.
- In the image below, which rays are refracted rays, and which are reflected rays?



Refraction and Snell's law

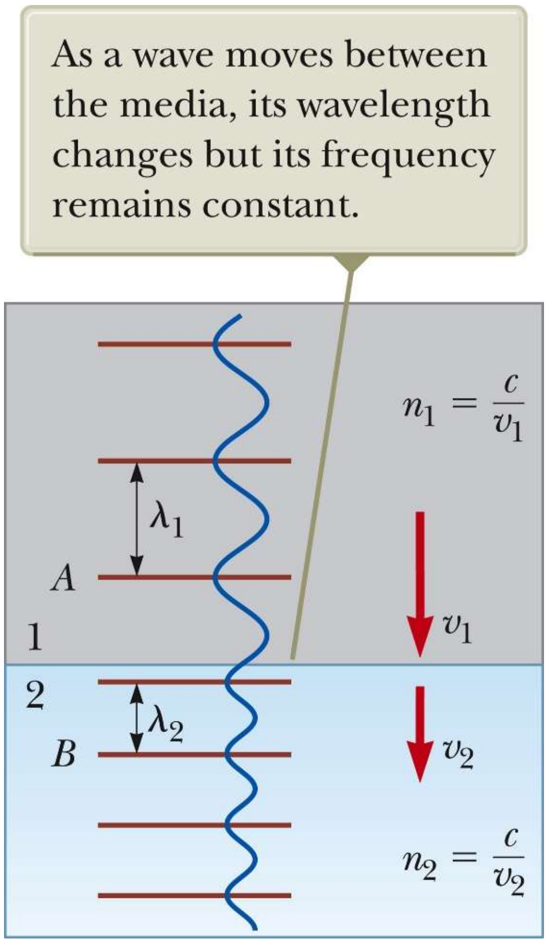


Snell's law: $$n_1 sin\theta_1 = n_2 sin\theta_2$$

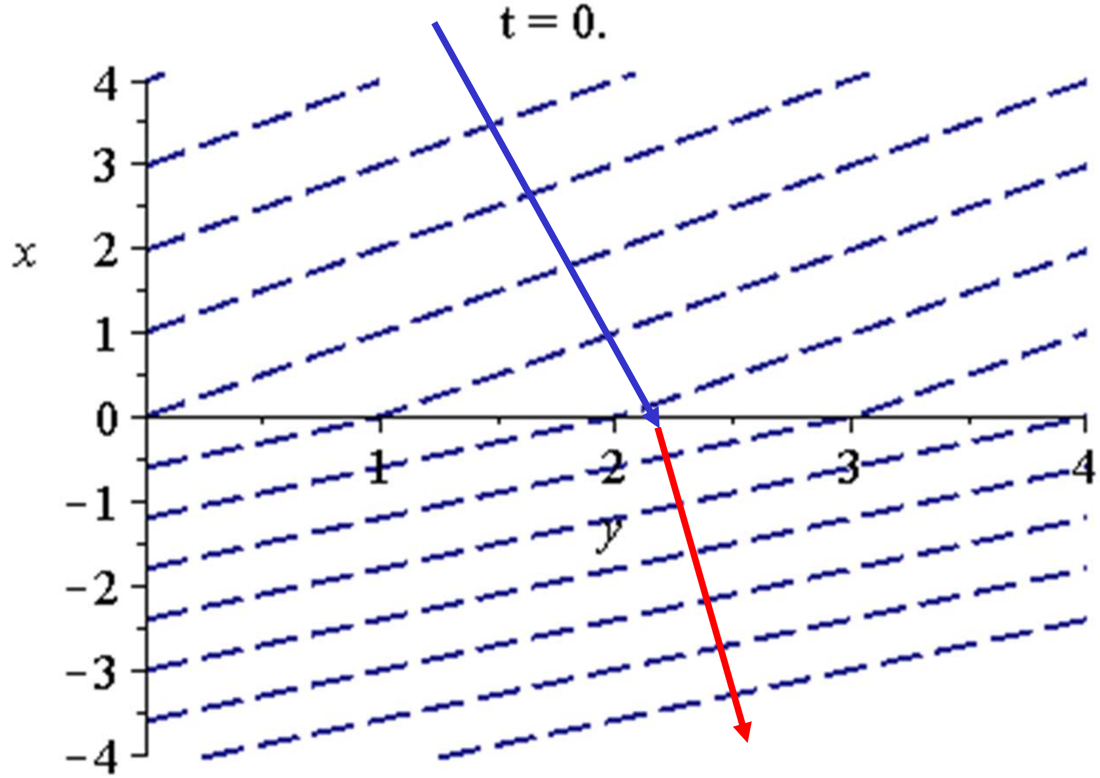
- When light moves into a higher index of refraction medium it slows down and bends toward the normal.
- When light moves into a lower index of refraction medium is speeds up and bends away from the normal.

(Derivation in a bit from Huygens's principle).

Snell's Law: Illustration



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



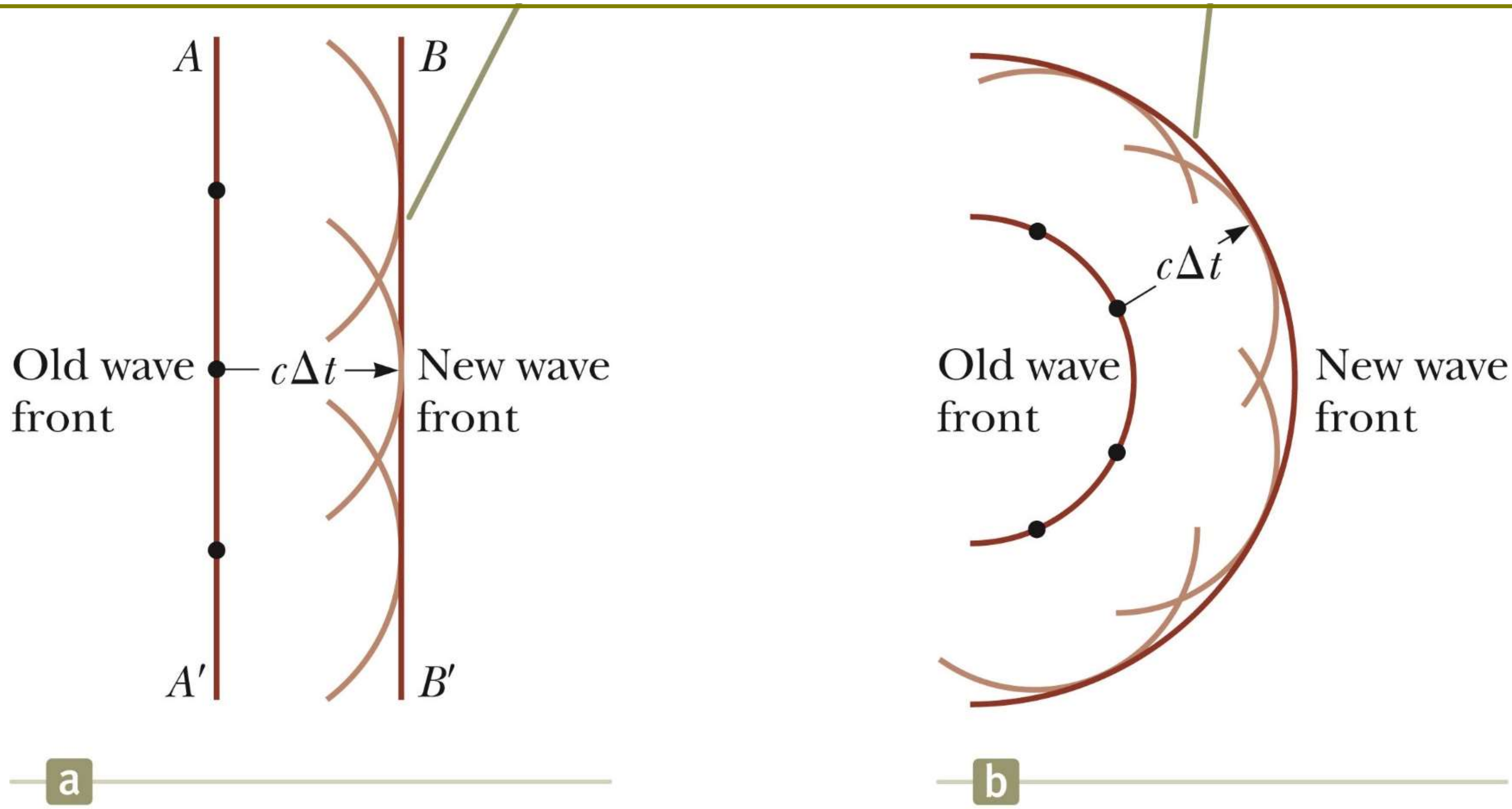
When light moves into an optically denser medium (higher index of refraction):

- The wavelength shortens (the frequency stays constant)
- The light bends toward the normal
- The speed of light gets reduced

Vice versa for moving into less dense medium (lower index of refraction).

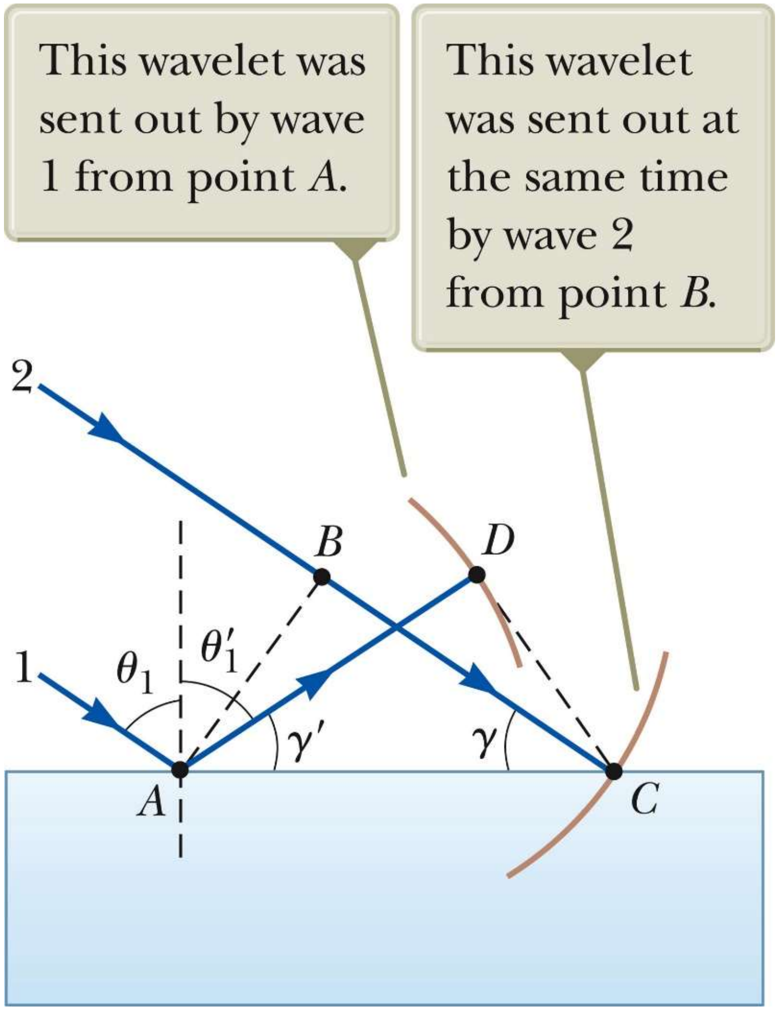
Huygens's principle

All points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, that propagate outward through a medium with speeds characteristic of waves in that medium. After some time has passed, the new position of the wave front is the surface tangent to the wavelets

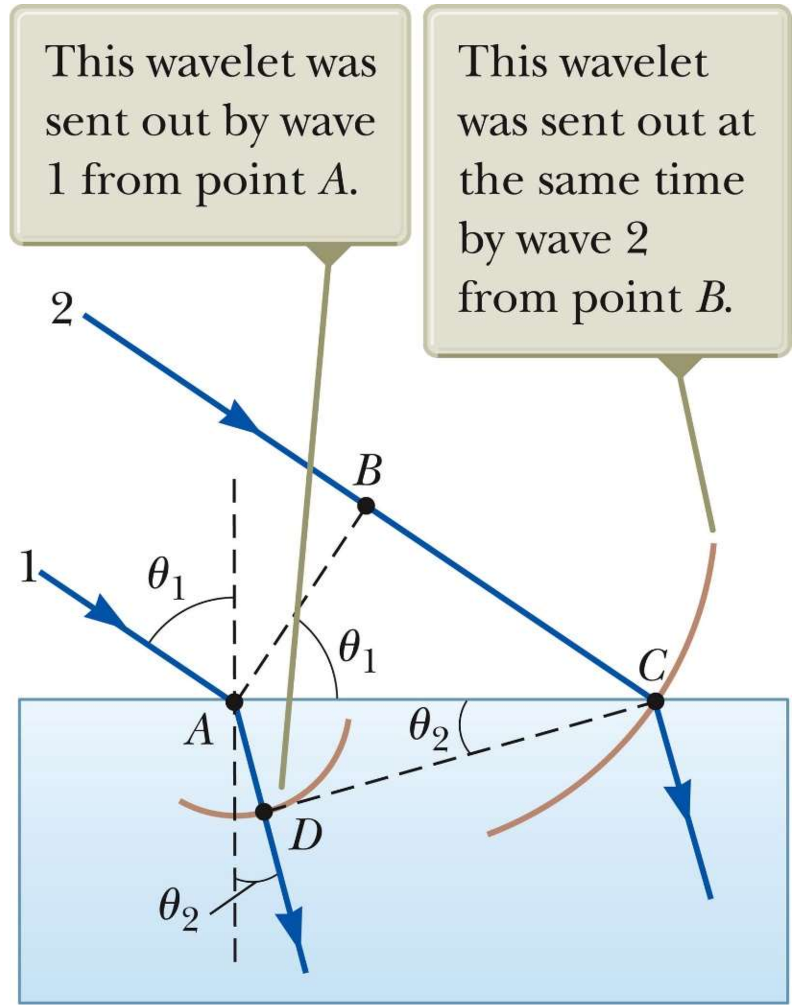


Huygens's principle

Law of reflection



Snell's law

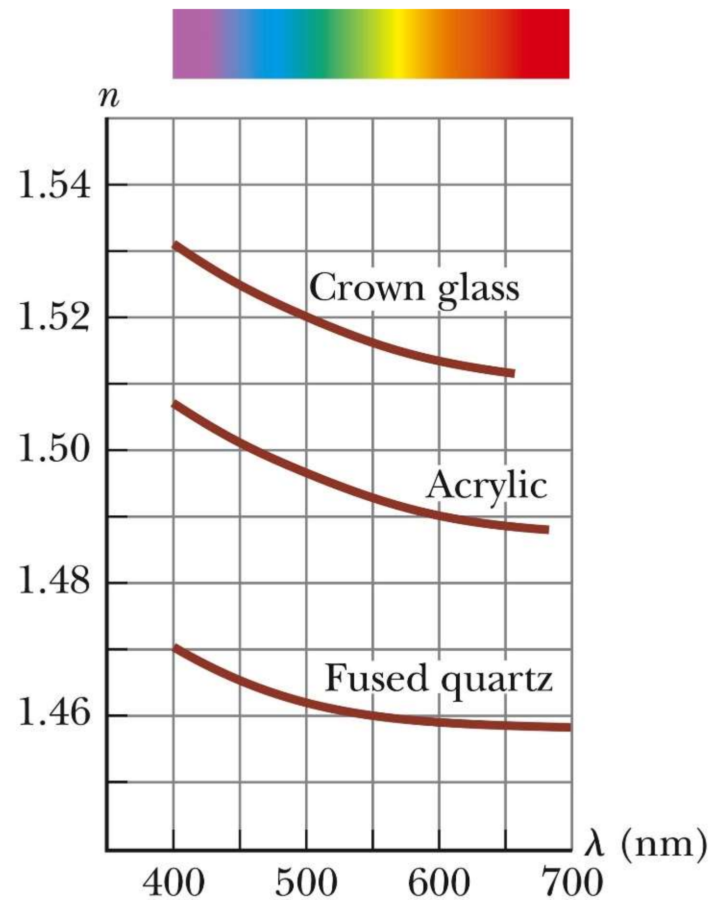


Dispersion

- The speed of light in a material can depend on frequency
  - Index of refraction *n* depends on frequency
  - Its dependence is usually given as a function of wavelength in vacuum
  - Called dispersion
- This means that different types of light bend by different amounts in any given material
- For most materials, the index of refraction is higher for short wavelengths

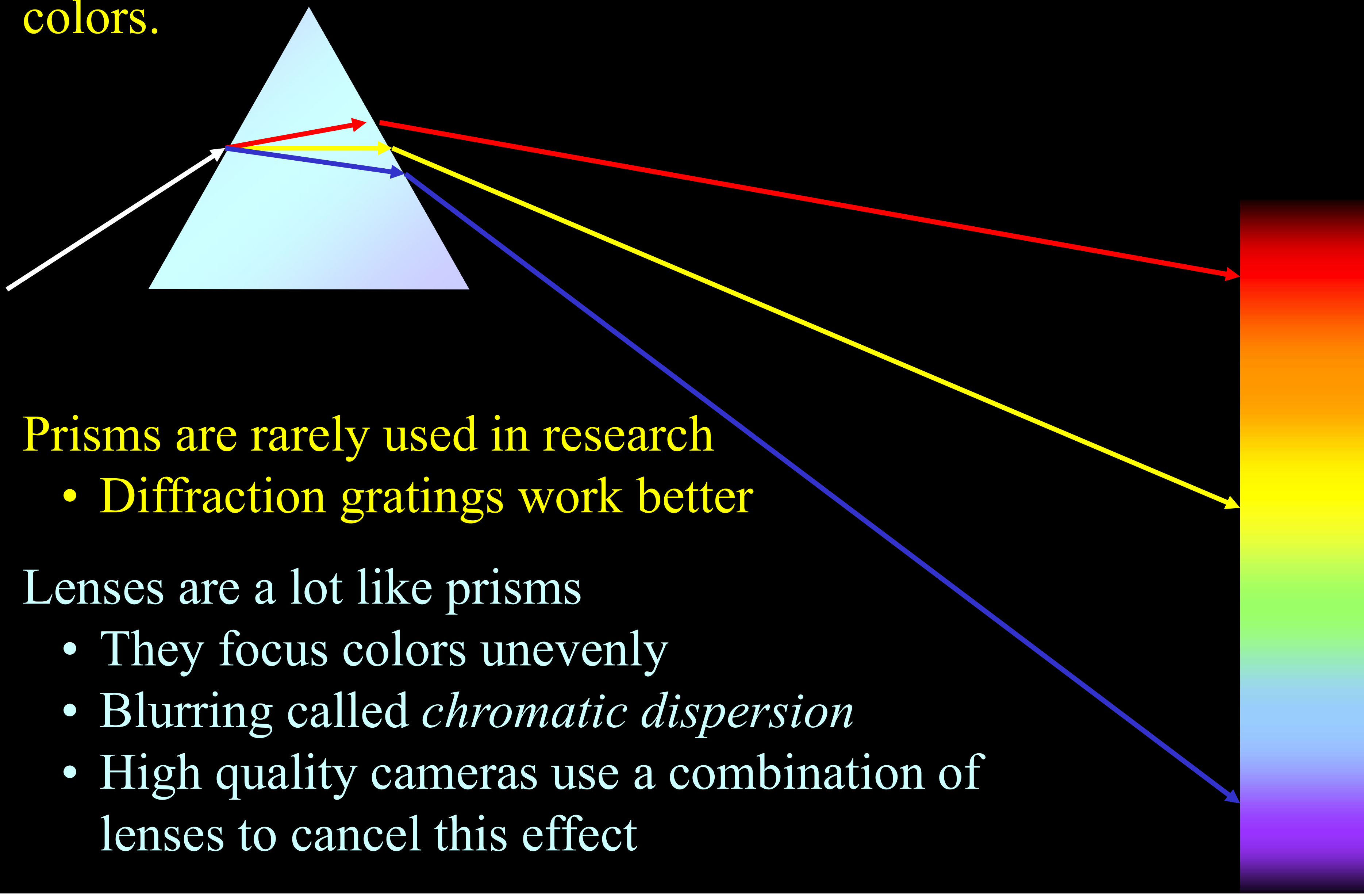
Red Refracts Rotten

Blue Bends Best



Prisms

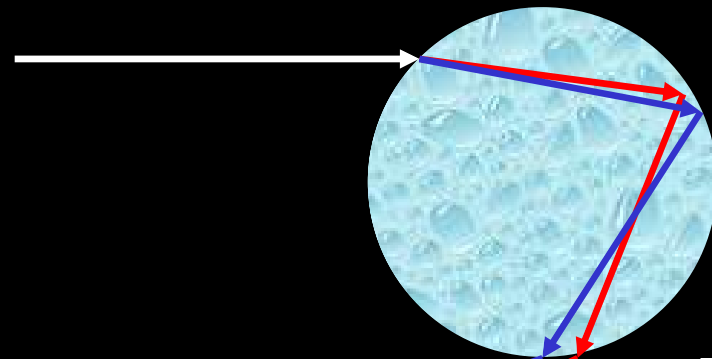
- Put a combination of many wavelengths (white light) into a triangular dispersive medium (like glass); light gets split into its individual colors.

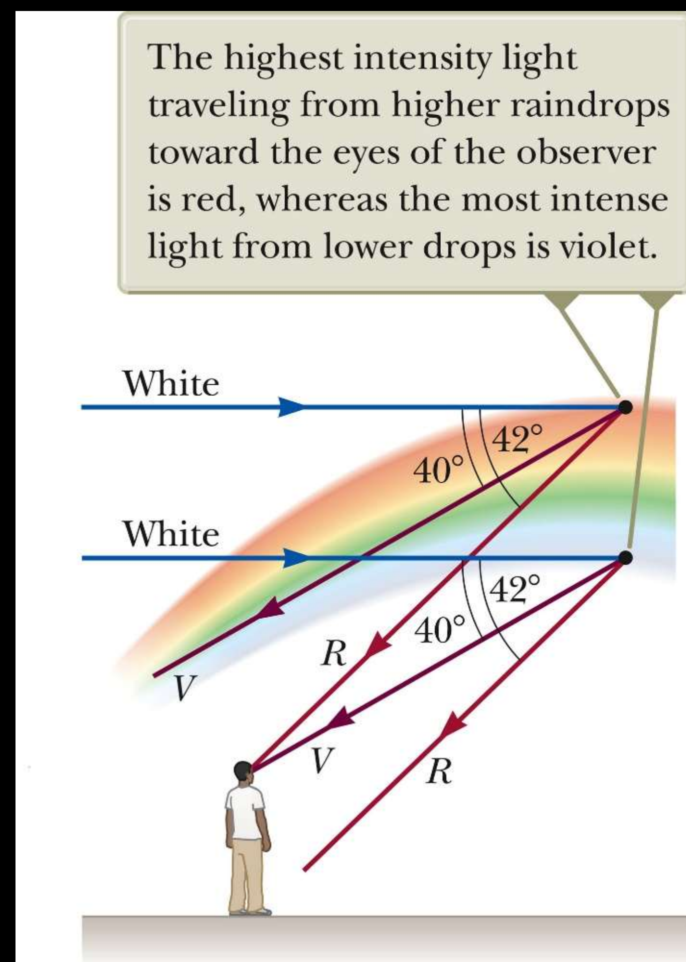


- Prisms are rarely used in research
  - Diffraction gratings work better
- Lenses are a lot like prisms
  - They focus colors unevenly
  - Blurring called *chromatic dispersion*
  - High quality cameras use a combination of lenses to cancel this effect

Rainbows

- A similar phenomenon occurs when light reflects off the inside of a spherical rain drop
- This causes rainbows
- If it reflects twice, you can get a double rainbow



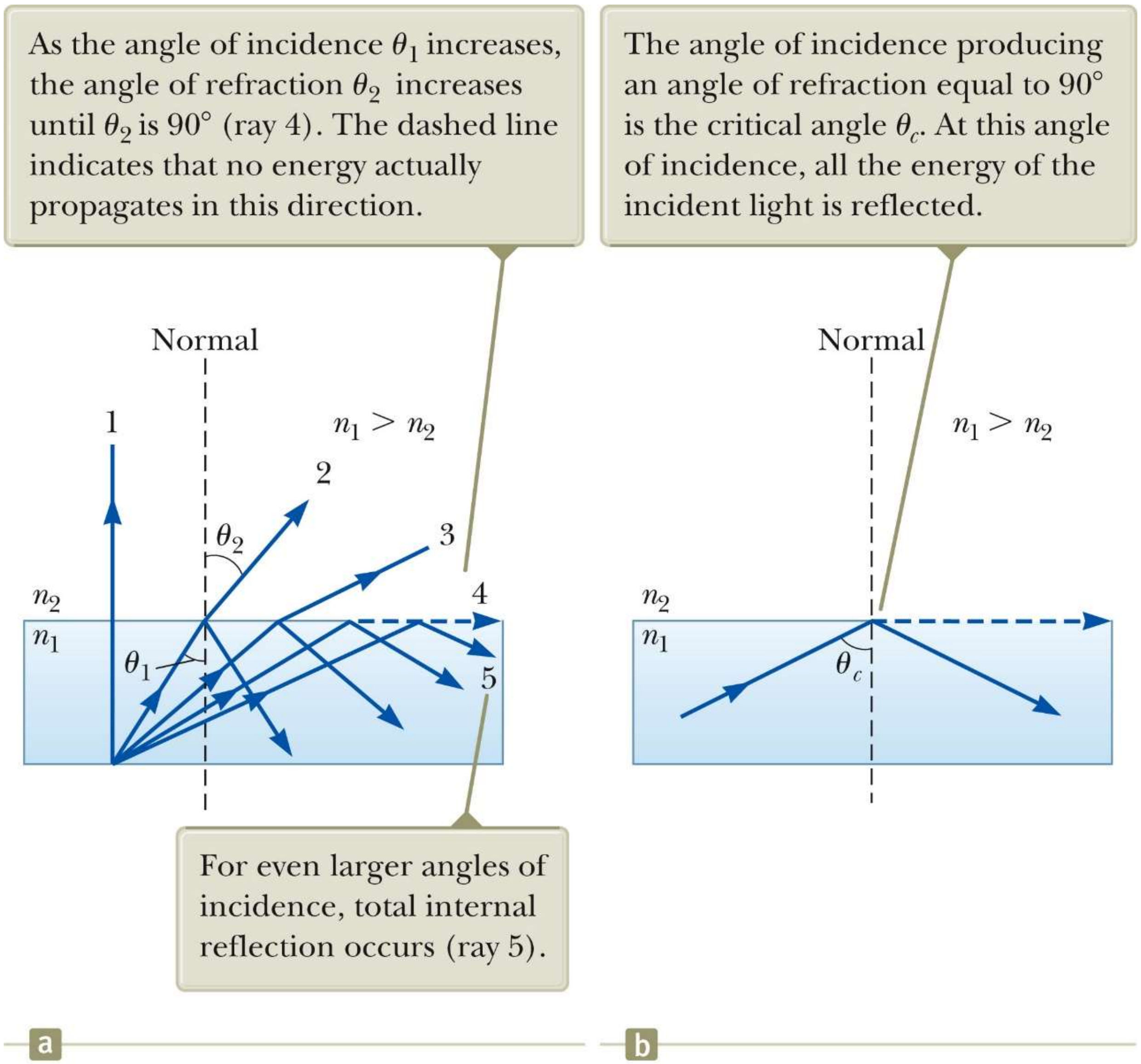


Red is bend the least and reaches the eye of the observer from higher up in the sky

Violet is bend the most and reaches the eye of the observer from lower in the sky.

Total Internal Reflection

When light hits the interface between a dense and a less dense medium at an angle larger than the critical angle, it will be totally internally reflected; it will never enter the less dense medium



$$\sin\theta_c = \frac{n_2}{n_1}$$

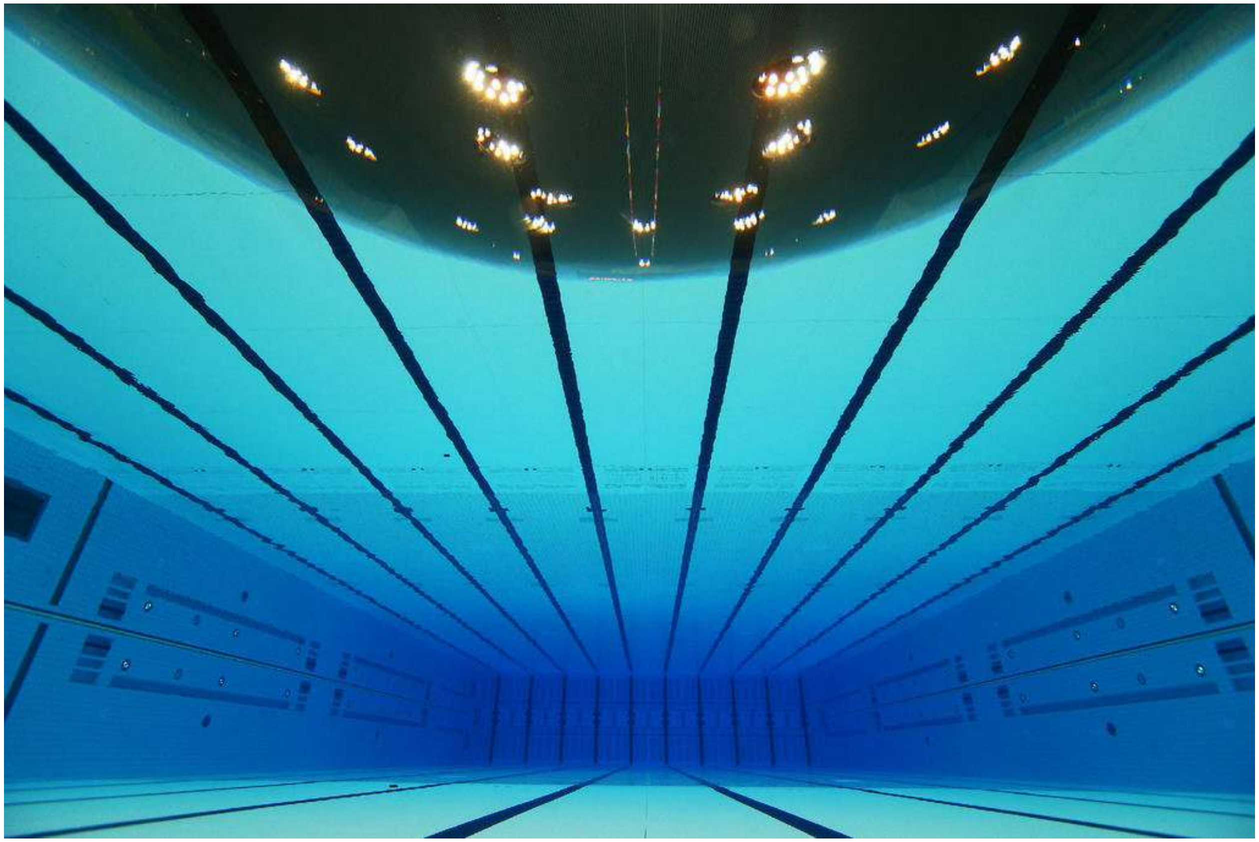
If $\theta_i > \theta_c$ we only get reflected light

How many of the five incident rays are totally internally reflected at the glass-air interface?

Total Internal Reflection

Test this out in the pool, and schematically explain the following image. When looking up from underneath the water:

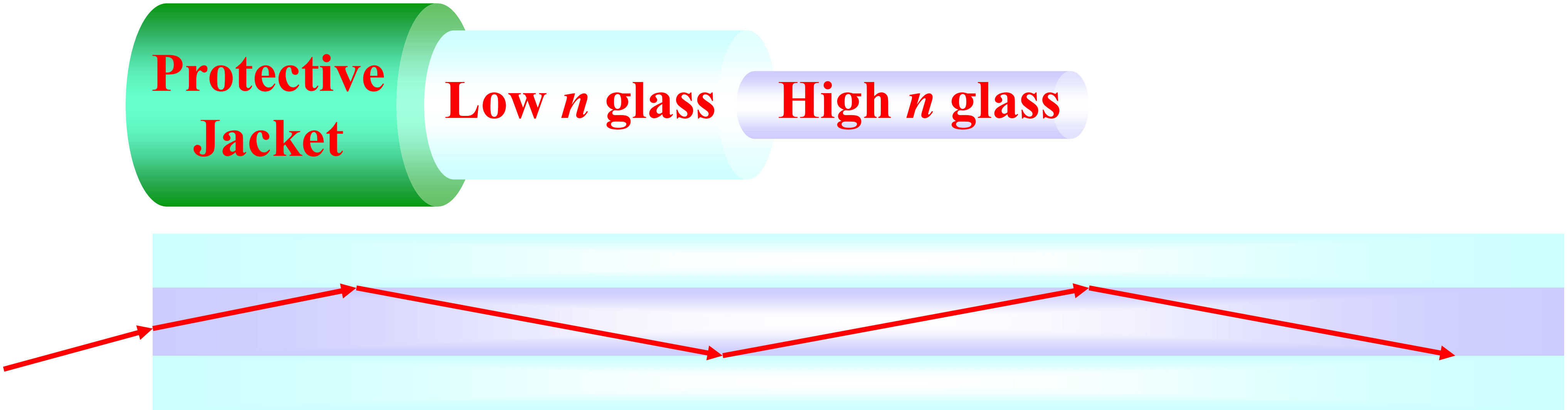
- If we look more in the forward direction ($\theta_i > \theta_c$), we see the bottom of the pool (total internal reflection)
- If we look more in the upward direction ($\theta_i < \theta_c$), we see the ceiling/sky (refraction)



White board example:

At what angle, θ_c, will the view change?

Total internal reflection application: Optical Fibers



- Light enters the high index of refraction glass
- It totally internally reflects – repeatedly
- Power can stay largely undiminished for many kilometers
- Used for many applications
  - Especially in transmitting signals in high-speed communications – up to 40 Gb/s

3.2. PHOTOMETRY

Introduction

Radiometry

is the measurement of optical radiation including visible light

Photometry

is the measurement of visible light only.

Angle

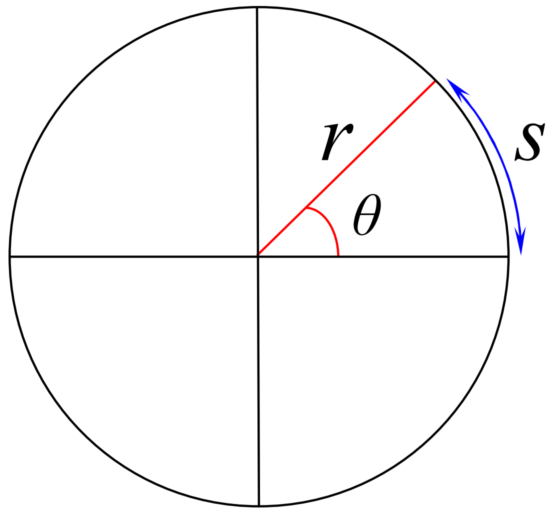
- Angle in two-dimension (2D) defined as

$$\theta = k\frac{s}{r}$$

where k is a proportionality constant and depends on the unit of measurement that is chosen.

for radian measure $k = 1$

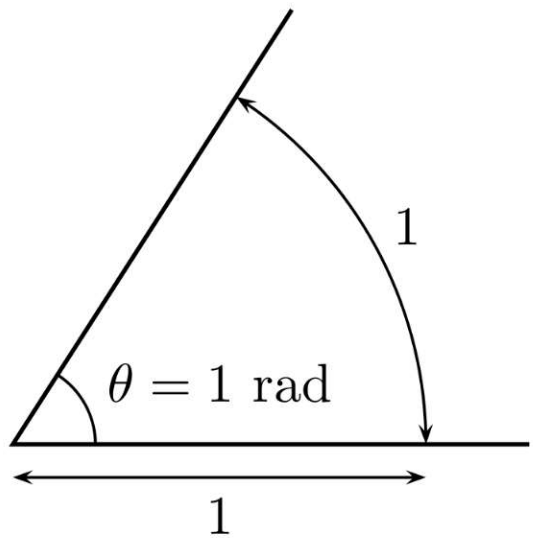
for degree measure $k = 180/\pi \approx 57.3$



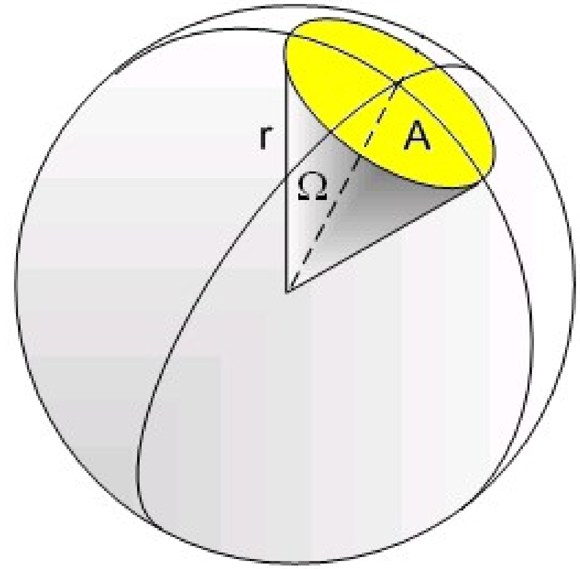
- Full circle is 2π radians:

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \ \text{rad}$$

- 1 radian defines an arc of a circle that has the same length as the circle's radius.
- 1 rad = 57.3°



Solid Angle



A : Surface area subtended from the center

r : Radius of the sphere

- The solid angle, Ω, is the 2D angle in 3D space that an object subtends at a point.
- Definition $$\Omega = \frac{A}{r^2}$$
- It is a measure of how large that object appears to an observer looking from that point.
- SI unit is steradian (sr)
- The solid angle of a sphere measured from a point in its interior is 4π sr.

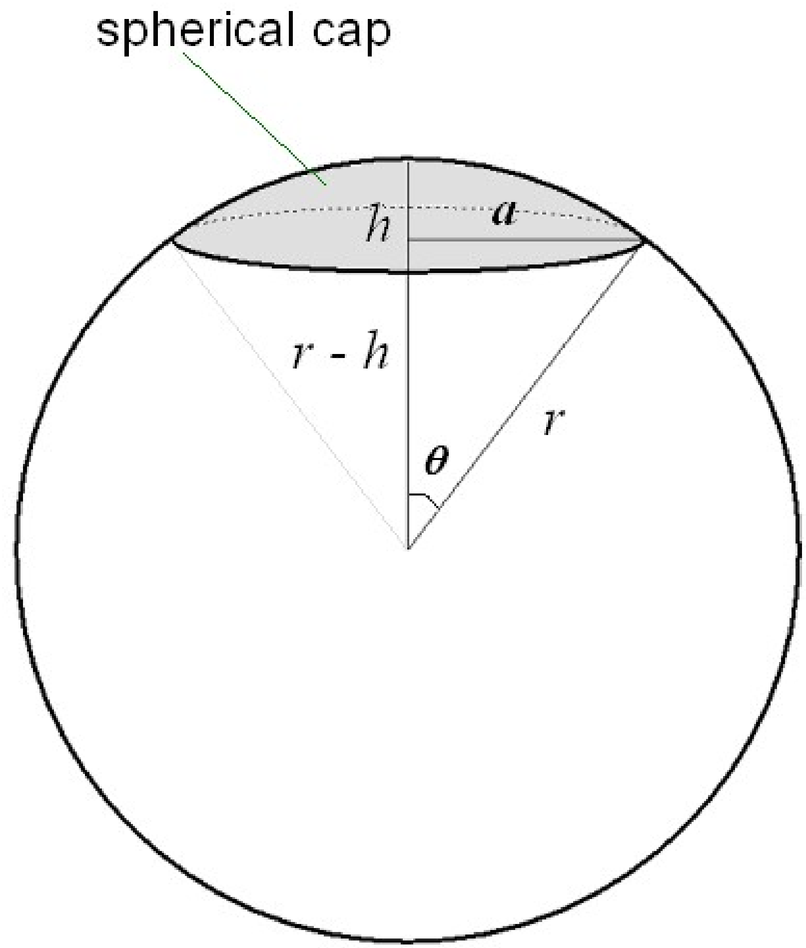
$$\Omega = \frac{A}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi \text{ sr}$$

- Area of a spherical cap:

$$A = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$$

- Solid angle subtended:

$$\Omega = \frac{A}{r^2} = 2\pi(1 - \cos\theta)$$



Radiometry

- Radiometry is the field that studies the measurement of electromagnetic radiation, including *visible light*.

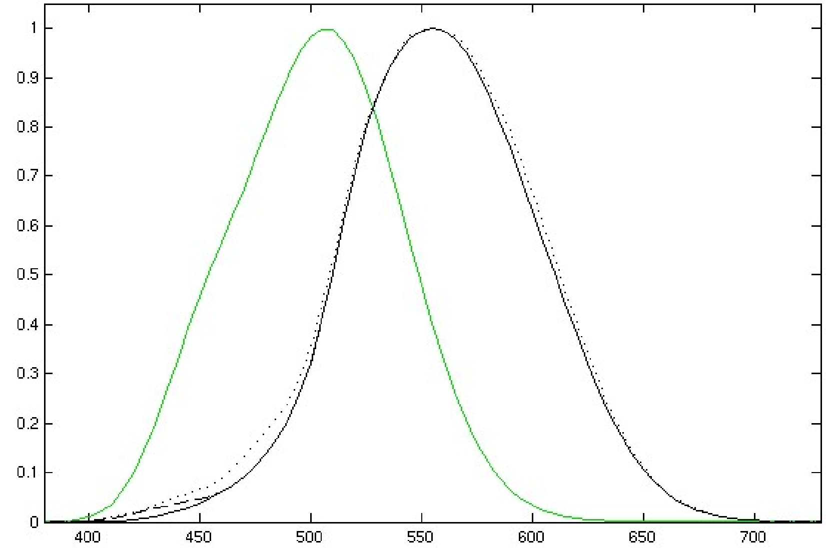
- Some SI radiometric units

Quantity	Symbol	SI unit	Abbr.
Radiant energy	Q	**Joule**	**J**
Radiant flux or Radiant power	Φ	**Watt**	**W**
Radiant intensity	I	**Watt per steradian**	**W/sr**
Irradiance	E	**Watt per square-meter**	W/m^2
Radiance	L	**Watt per steradian per meter-square**	$W/sr.m^2$

Photometry

- Photometry is the science of the measurement of light, in terms of its perceived brightness to the human eye.

- The human eye is not equally sensitive to all wavelengths of visible light.



Photopic (black) and scotopic (green) luminosity functions.

- *For everyday light levels*, the **photopic** curve (black) best approximates the response of the human eye.
- *For low light levels*, the response of the human eye changes, and the **scotopic** curve (green) applies.

Radiometry and Photometry Conversion

The radiant power at each wavelength is weighted by a luminosity function

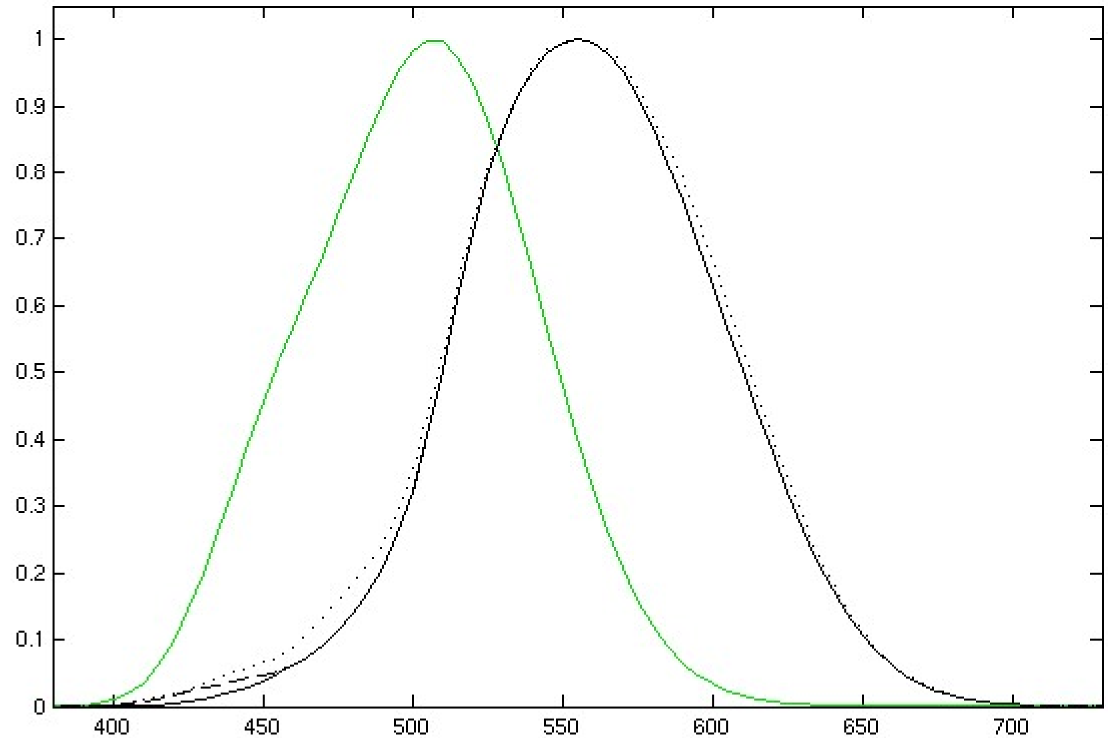
V(λ) that models human brightness sensitivity.

For photopic curve (black):

$$V(\lambda) = 1.019e^{-285.4(\lambda-0.559)^2}$$

For scotopic curve (green)

$$V'(\lambda) = 0.992e^{-312.9(\lambda-0.503)^2}$$



- Some SI photometric units

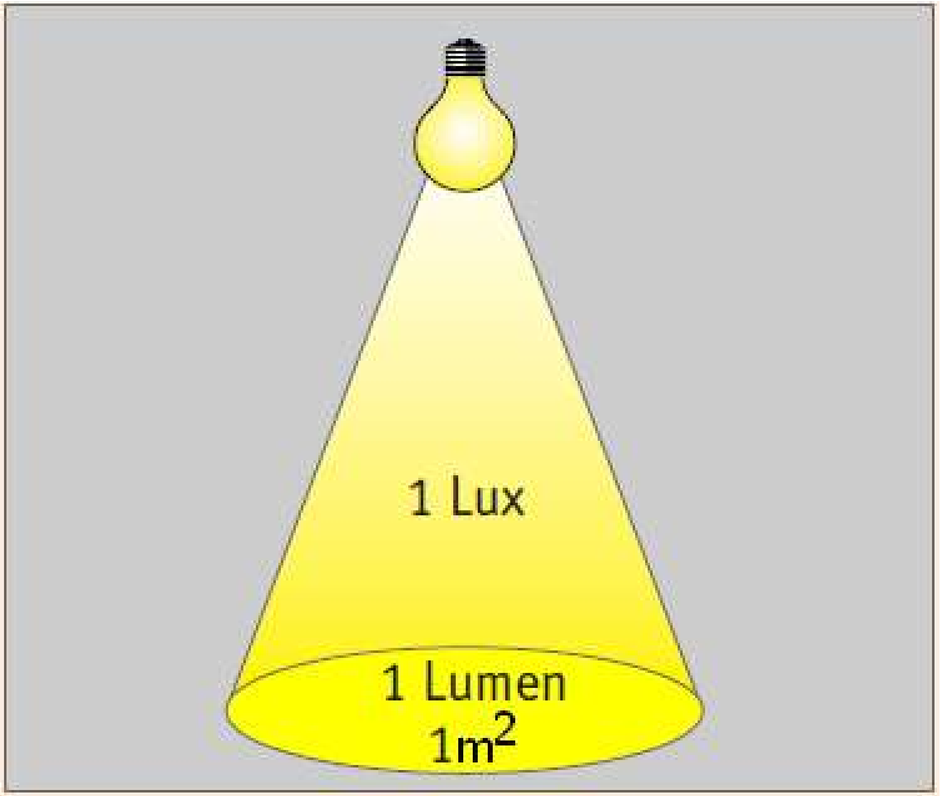
Quantity	Symbol	SI unit	Abbr.
Luminous energy	Q_v	**lumen.second**	**lm.s**
Luminous flux or Luminous power	Φ_v	**lumen**	**lm**
Luminous intensity	I_v	**candela**	**cd = lm/sr**
illuminance	E_v	**lumen per meter-square**	**lux = lm/m^2**
Luminance	L_v	**lumen per steradian per meter-square**	**lm/sr.m^2 = cd/m^2**

- **Typical illuminaces:**

*** Direct sun light 100,000 lux**

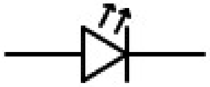
*** Working desk 500 lux**

*** Hospital corridors 20-50 lux**

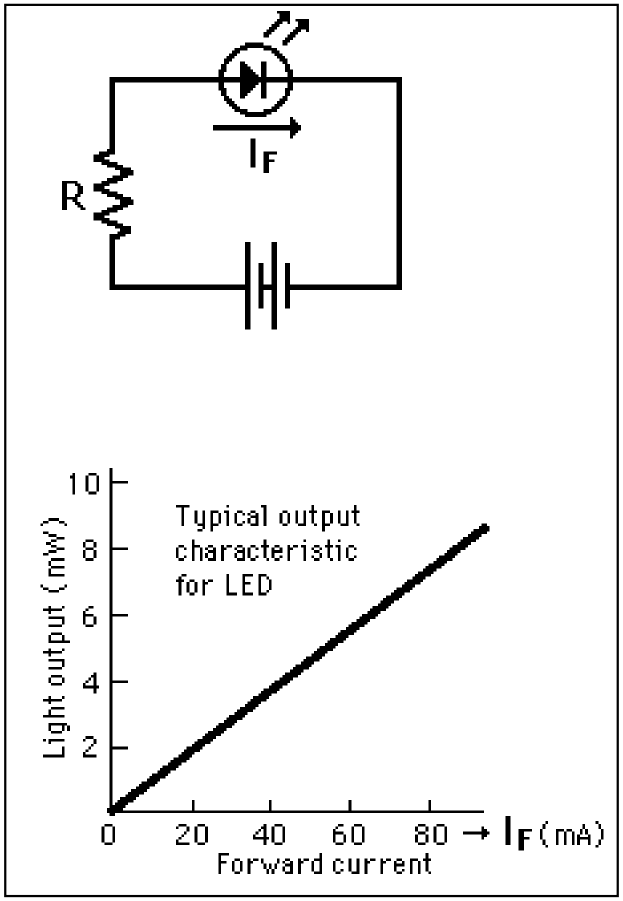


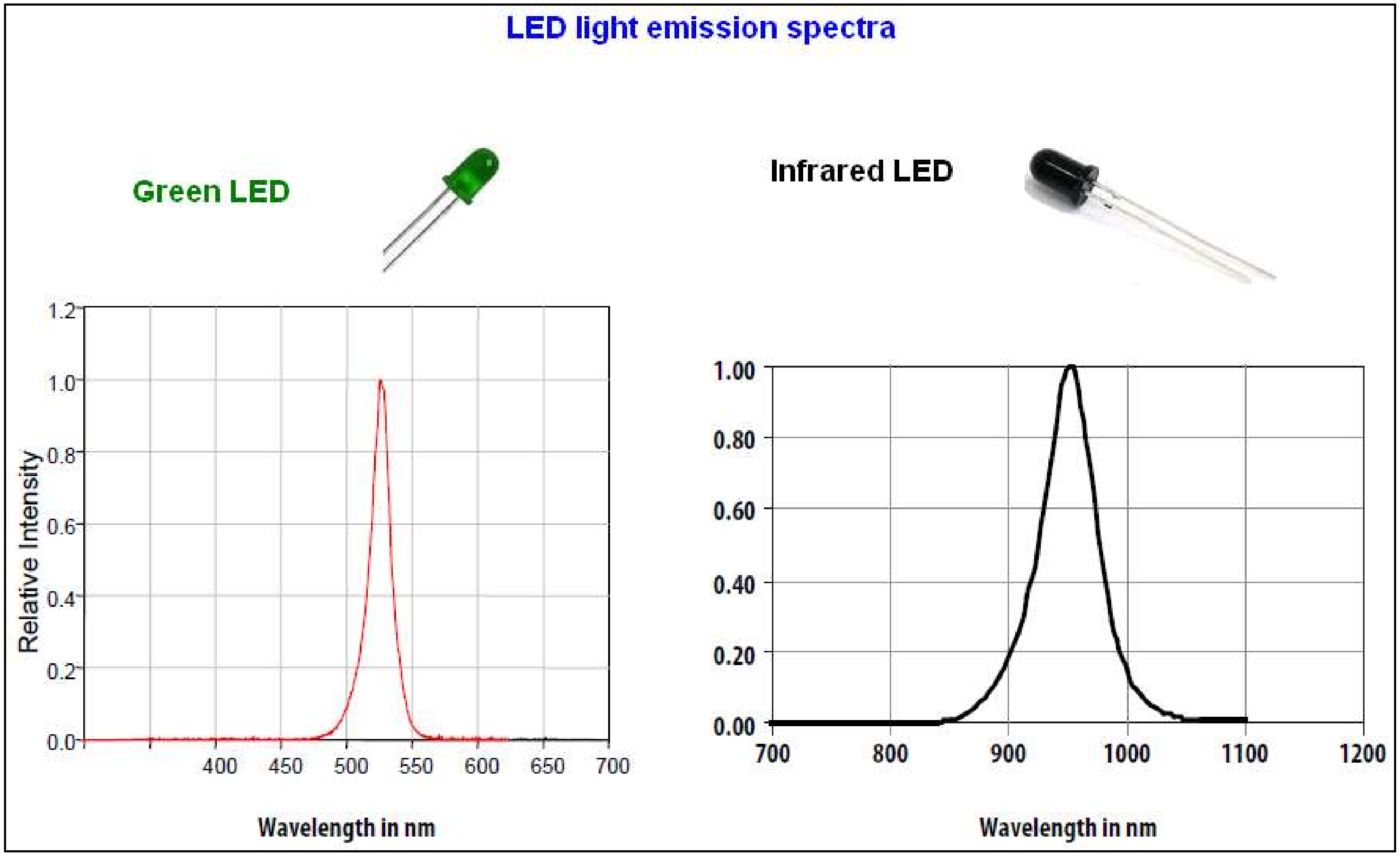
$1\,lx = 1\,lm/m^2$

LED

- A light-emitting diode (LED) is a semiconductor light source.
- Circuit Symbol: 

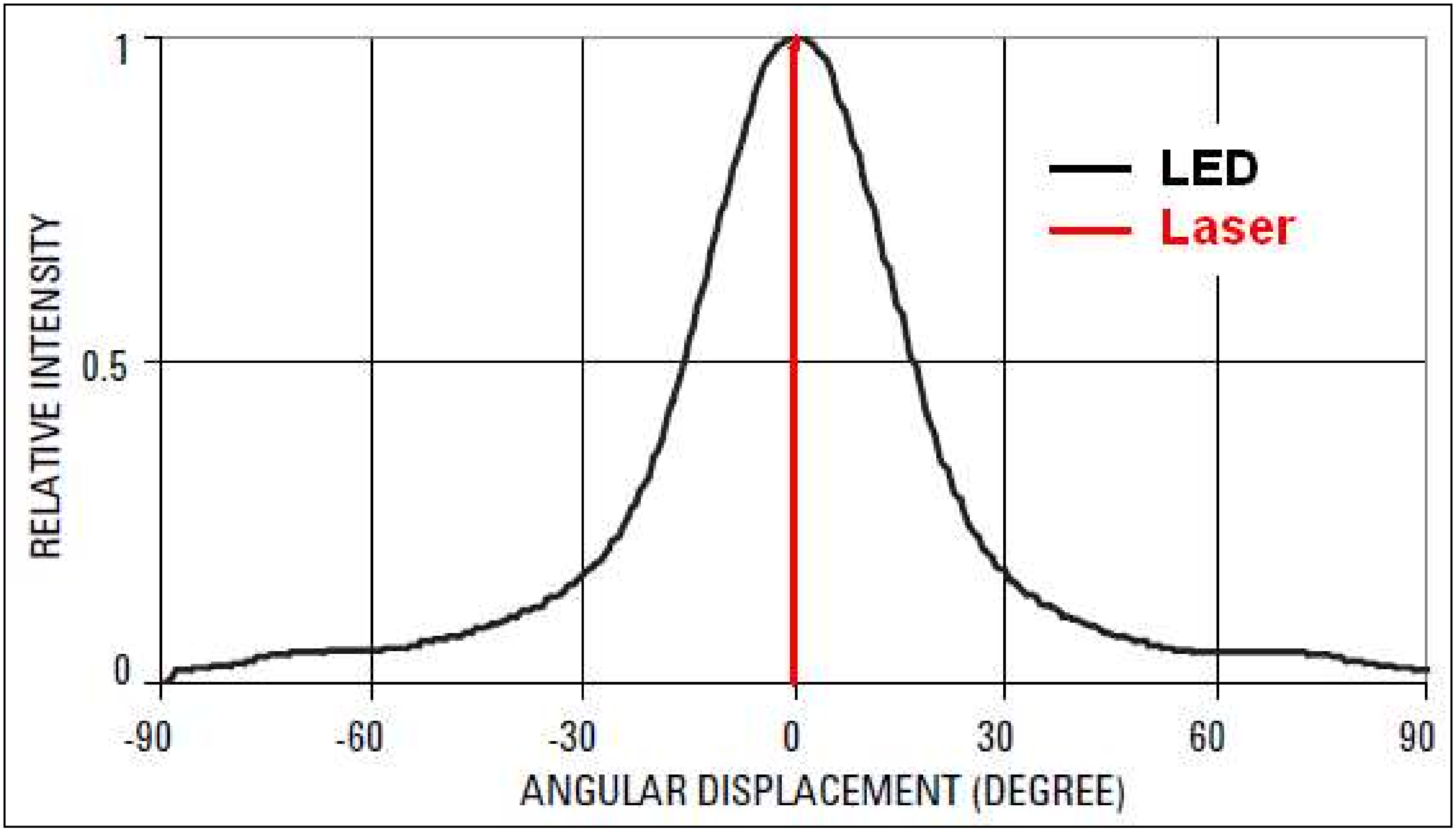
- A LED can produce the visible, ultraviolet and infrared wavelengths, with very high brightness.





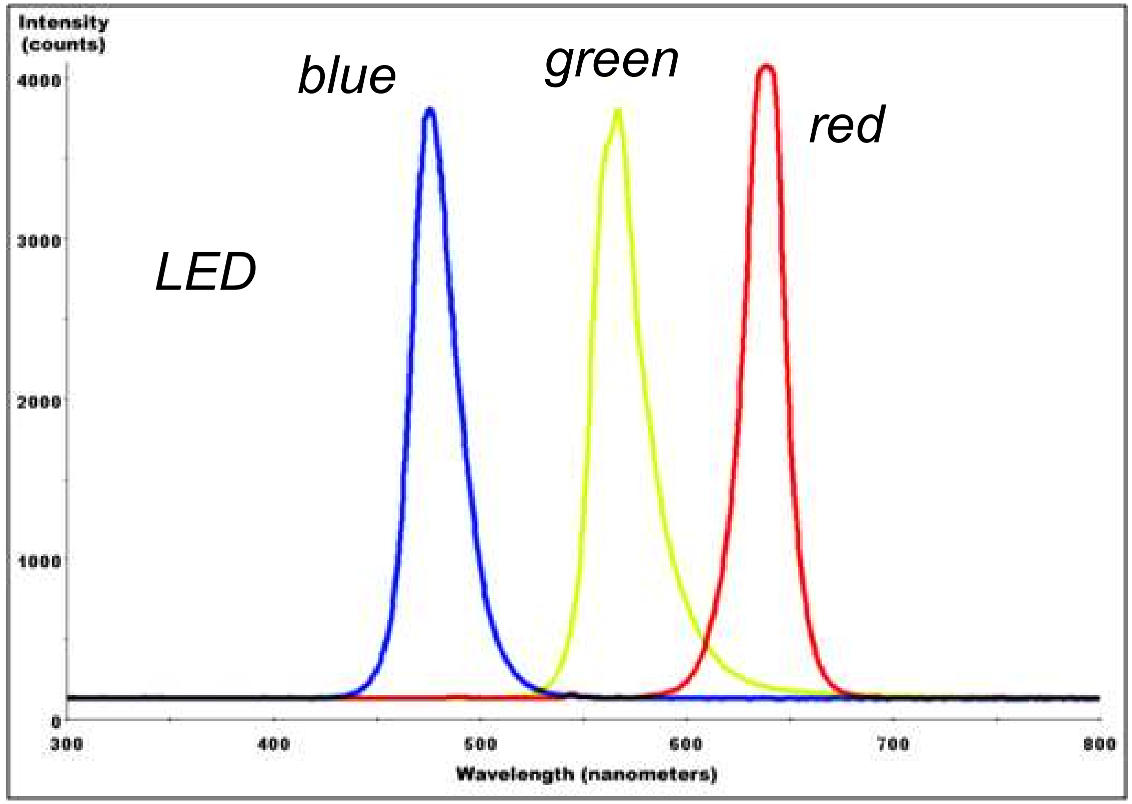
LED vs Laser

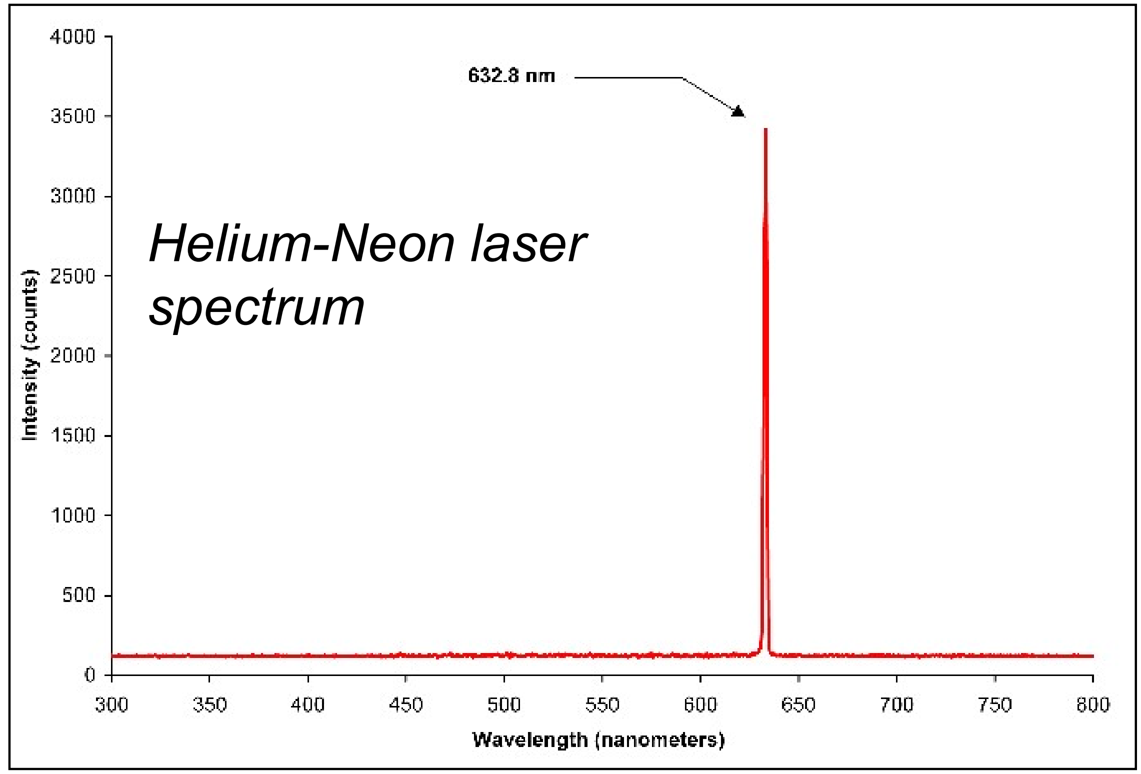
- Comparison of Beam Divergence of LED and Laser



LED vs Laser

- Spectral width of the laser is 10,000 times narrower than the spectral width of a light-emitting diode.





Unit Comparison

Radiometry measures the entire radiant power and quantities derived from it

Photometry measures that part of radiant power perceived by human eye as light

source power	
radiant flux	luminous flux
Watt (W)	Lumen (lm)

looking at illuminated surface

power per unit area	
irradiance	illuminance
W/m^2	lm/m^2 = lux

looking at source

power per area per solid angle	
radiance	luminance
$W/sr.m^2$	$lm/sr.m^2 = cd/m^2$

power per unit solid angle	
radiant intensity	luminous intensity
W/sr	lm/sr = cd

Photometer

- Photometer is an instrument for measuring **light intensity**.
- Most of the, photometers are used to measure iluminance (E_v) or irradiance (E).
- Measuring E_v is important in *illumination Engineering.*
- Most photometers detect the light with photoresistors, photodiodes or photomultipliers (*we will see later*).

Equations: Luminous Flux

- We know from the definition of the candela that there are 683 lumens per watt at a wavelength 555 nm (in vacuum or air). This is the

- The conversion from watts to lumens at any other wavelength involves the product of the power (watts) and the V(λ) value at the wavelength of interest. For *mono-chromatic wave* we can use

$$\Phi_{\mathrm{v}} = (683\,\mathrm{lm/W})\,\Phi V(\lambda)$$

- In order to convert a source with *non-monochromatic* spectral distribution to a luminous quantity, the situation is decidedly more complex. We must know the spectral nature of the source, because it is used in an equation of the form:

$$\Phi_{\mathrm{v}} = (683\,\mathrm{lm/W})\int_0^{\infty} \Phi(\lambda)\,V(\lambda)\,d\lambda$$

EXAMPLE

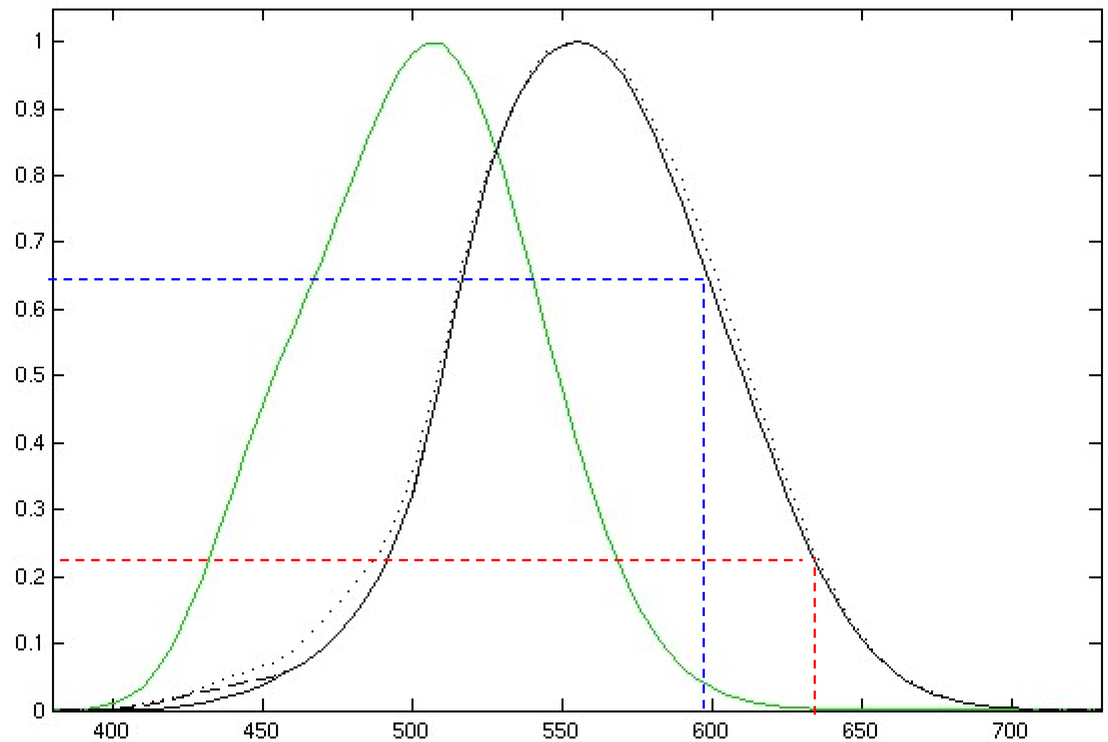
Compare brightness' of two 5 mW laser pointers at 635 nm and 600 nm.

SOLUTION

* at λ =600 nm, V(λ) = 0.650 --> $$\Phi_v = (683\,\frac{\text{lm}}{\text{W}})(0.005\text{ W})(0.65) = 2.22\ \text{lm}$$

* at λ = 635 nm, V(λ) = 0.217 --> $$\Phi_v = (683\,\frac{\text{lm}}{\text{W}})(0.005\text{ W})(0.217) = 0.74\ \text{lm}$$

The shorter wavelength (600 nm) laser pointer will create a spot that is almost 3 times as bright as the longer wavelength (635 nm) laser assuming the same beam diameter.

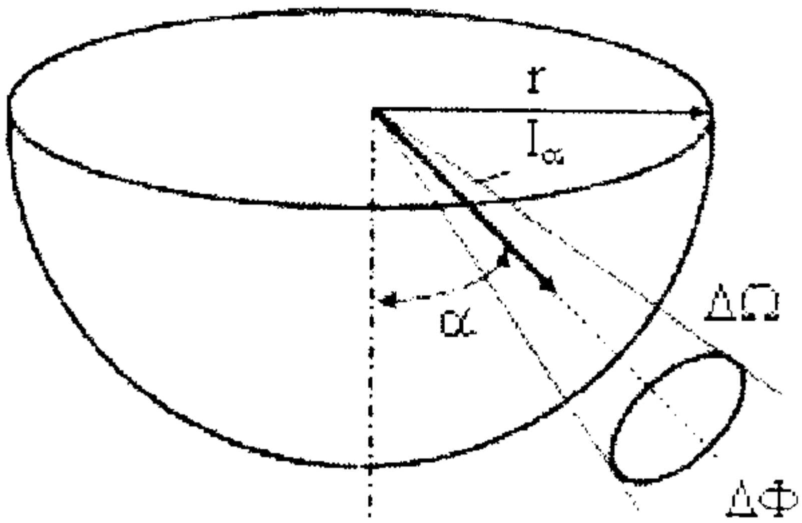


Equations: Luminous Intensity

This is defined for a point source and has both magnitude and direction.

Average calculation:

$$I_{\mathrm{AVR}} = \frac{\Delta\Phi_{\mathrm{v}}}{\Delta\Omega}$$



Differential limit:

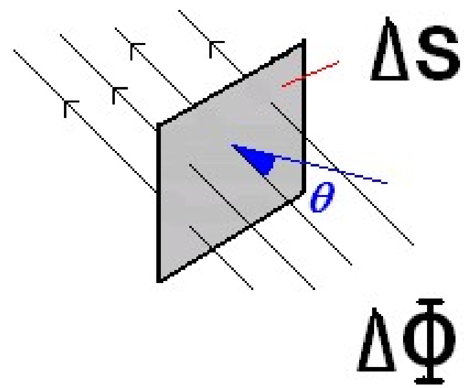
$$I_{\alpha} = \frac{d\Phi_{\mathrm{v}}}{d\Omega}$$

Unit : Candela (cd)

Equations: Illuminance

Average calculation:

$$E_{\mathrm{AVR}} = \frac{\Delta\Phi_{\mathrm{v,perp.}}}{\Delta S} = \frac{\Delta\Phi_{\mathrm{v}}}{\Delta S}\cos\alpha$$



Differential limit:

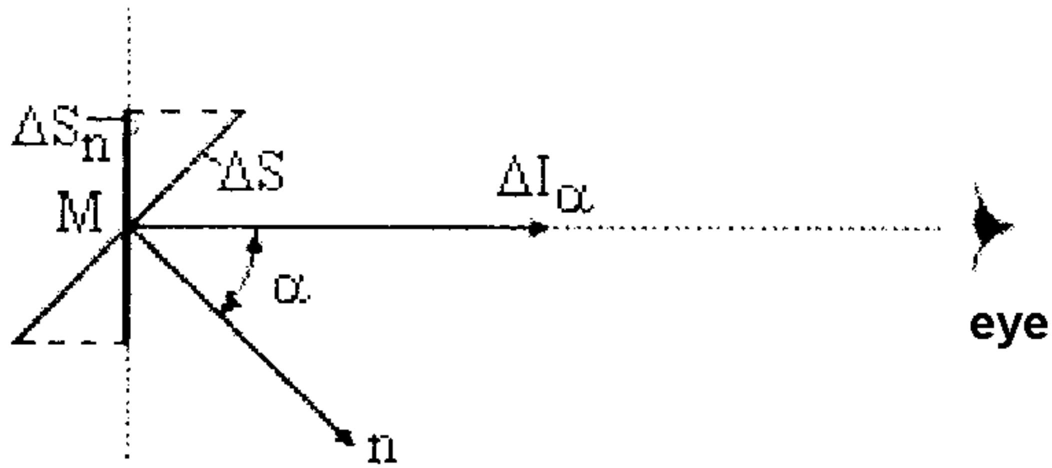
$$E = \frac{d\Phi_{\mathrm{v}}}{dS}\cos\alpha$$

Unit: Lux

Equations: Luminance

Average calculation:

$$L_{\mathrm{AVR}} = \frac{\Delta I_{\alpha}}{\Delta S_{n}}$$



Differential limit:

$$L = \frac{dI_{\alpha}}{dS_{n}}$$

Unit: cd/m^2

Equations

Radiative flux of point source:

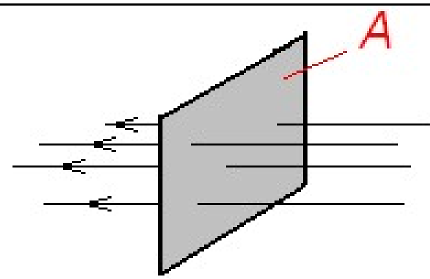
$$\Phi = 4\pi I$$

Luminous flux of point source:

$$\Phi_{\mathrm{v}} = 4\pi I_{\mathrm{v}}$$

Irradiance on area *A*:

$$E = \frac{\Phi}{A}$$

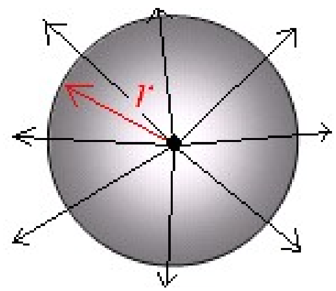


illuminance on area *A*:

$$E_v = \frac{\Phi_{\mathrm{v}}}{A}$$

Irradiance of a point source of intensity *I*

$$E = \frac{\Phi}{A} = \frac{4\pi I}{4\pi r^2} = \frac{I}{r^2}$$

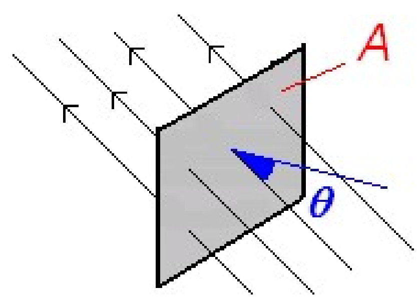


illuminance of a point source of intensity I_{v}

$$E_v = \frac{\Phi_{\mathrm{v}}}{A} = \frac{4\pi I_{\mathrm{v}}}{4\pi r^2} = \frac{I_{\mathrm{v}}}{r^2}$$

If radiation direction makes an angle θ with the normal of irradiated surface

$$E = \frac{I}{r^2}\cos\theta$$



If radiation direction makes an angle θ with the normal of illuminated surface

$$E_v = \frac{I_{\mathrm{v}}}{r^2}\cos\theta$$

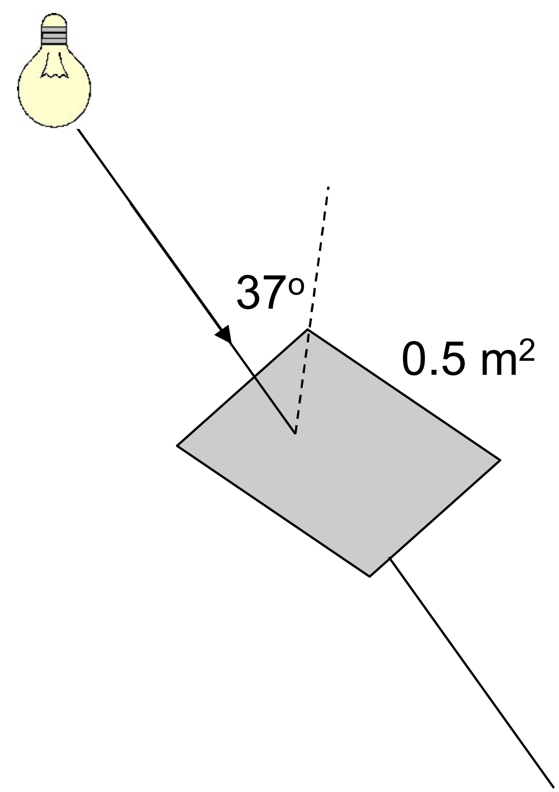
EXAMPLE

The light rays emerging from a point source of intensity 100 cd fall on a planar surface whose area is 0.5 m^2 at distance 1 m from the source. The rays make an angle of 37^o with the normal of a planar surface.

(a) Find the total flux of the source.

(b) Find the illuminance on the surface.

(c) Find the flux on the surface.



SOLUTION

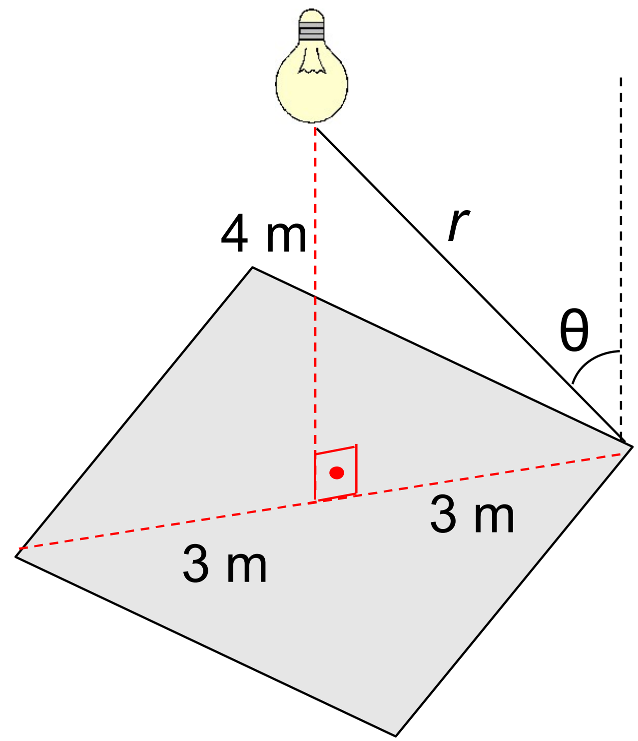
(a) $$\Phi_v = 4\pi I_v = (4\pi)(100\ \text{cd}) = 1256.6\ \text{lm}$$

(b) $$E_v = \frac{I_v}{r^2}\cos\theta = \frac{100\ \text{cd}}{(1\ \text{m})^2}\cos 37^o = 80.0\ \text{lux}$$

(c) $$\Phi_v = E_v A = (80.0\ \text{lux})(0.5\ \text{m}^2) = 40.0\ \text{lm}$$

EXAMPLE

A 1000 cd-bulb is hang at a height of 4 m from the center of the floor of a room having square shape with diagonal length of 6 m as shown in figure. Calculate the illuminance of the bulb at any corner of the floor.



SOLUTION

From figure

$$r = \sqrt{3^2 + 4^2} = 5.0\ \text{m}$$

$$\cos\theta = \frac{4}{5} = 0.8$$

illuminance at any corner:

$$E_v = \frac{I_v}{r^2}\cos\theta = \frac{1000\ \text{cd}}{(5\ \text{m})^2}(0.8) = 32.0\ \text{lux}$$

Efficiency and Efficacy of a Light Source

- Efficiency and efficacy may be defined as follows*

$$\text{Efficiency} = \frac{\text{Visible Radiant Flux}}{\text{Power Consumed}} = \frac{\Phi}{P} \longrightarrow \frac{\text{Watts}}{\text{Watts}} \equiv \text{unitless}$$

$$\text{Efficacy} = \frac{\text{Luminous Flux}}{\text{Radiant Flux}} = \frac{\Phi_v}{\Phi} \longrightarrow \frac{\text{lumens}}{\text{Watts}} \equiv \frac{\text{lm}}{\text{W}}$$

** Depending on context, the "power" can be either*

the radiant flux of the source's output, or

the total electric power consumed by the source.

Type	Luminous Efficiency	Luminous Efficacy (lm/W)
Sun	12 %	80
100–200 W tungsten lamb	2 %	13 – 15
10–30 W Fluorescent lamb	8 % – 11%	46 – 75
White LED	1 % – 22 %	5 – 150
Ideal monochromatic 555 nm source	100 %	683

EXAMPLE

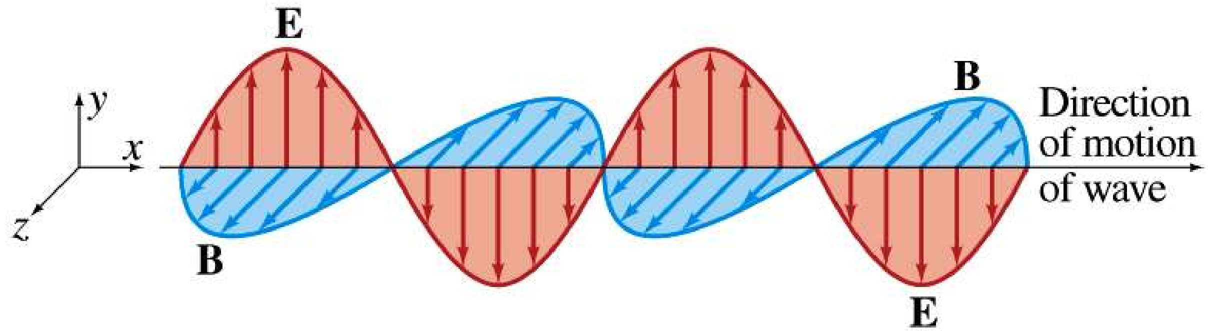
On a table, one needs a 60 lux illuminance.
A 40 W-lamp whose luminous efficacy is 100 lm/W and efficiency is 20% will be used for illumination. Calculate the height of the lamb that must be hang from the table.

SOLUTION

Total flux of the lamb: $\Phi_v = (100\,\frac{\text{lm}}{\text{W}})(40\text{ W})(0.2) = 800.0\text{ lm}$

The luminous intensity of the bulb: $I_v = \frac{\Phi_v}{4\pi} = \frac{800.0\text{ lm}}{4\pi} = 63.7\text{ cd}$

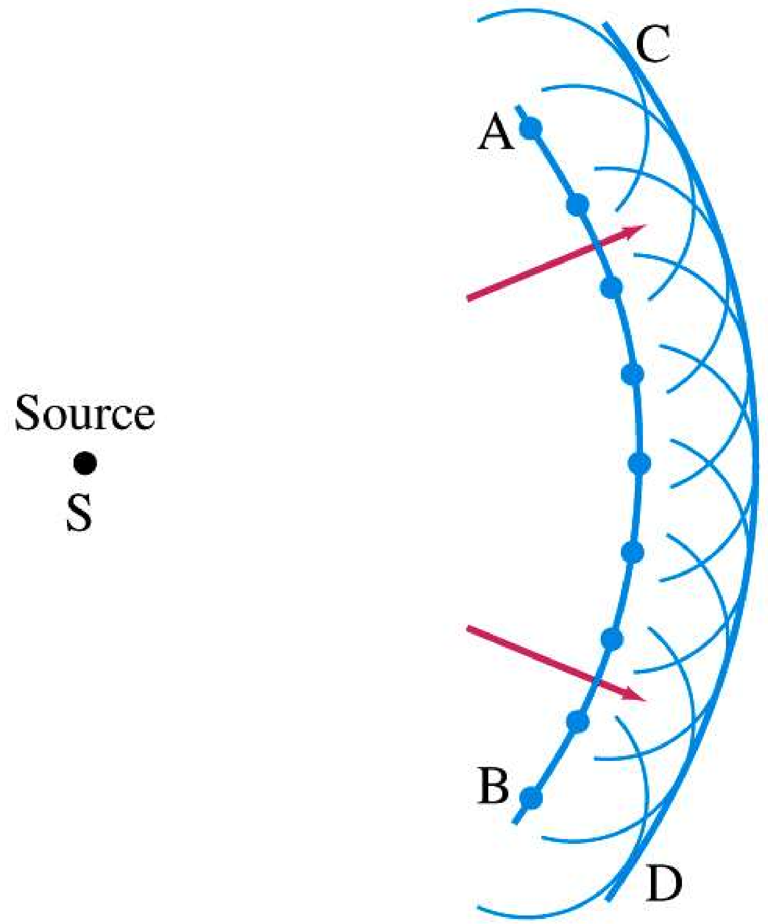
From: $I_v = \frac{E_v}{d^2} \longrightarrow d = \sqrt{\frac{E_v}{I_v}} = \sqrt{\frac{60\text{ lux}}{63.7\text{ cd}}} = 0.97\text{ m} \approx 1.0\text{ m}$



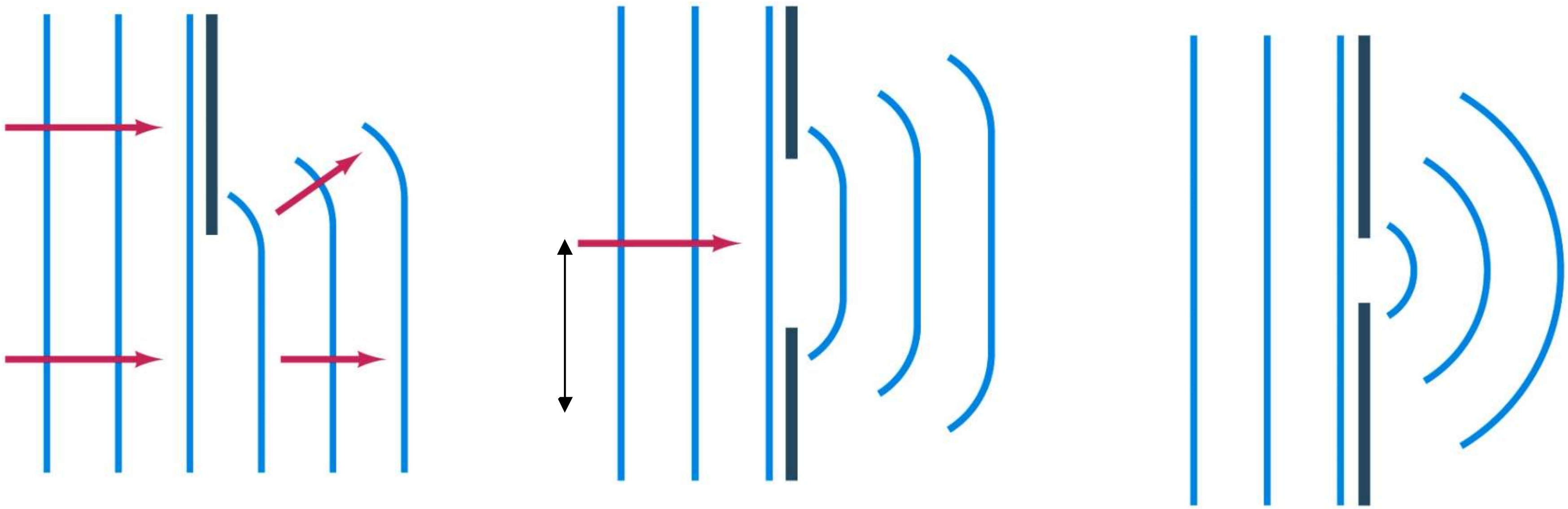
3.3. INTERFERENCE IN LIGHT WAVES

Huygens’ Principle

- Every point on a propagating wavefront serves as the source of spherical wavelets, such that the wavelets at sometime later is the envelope of these wavelets.
- If a propagating wave has a particular frequency and speed, the secondary wavelets have that same frequency and speed.

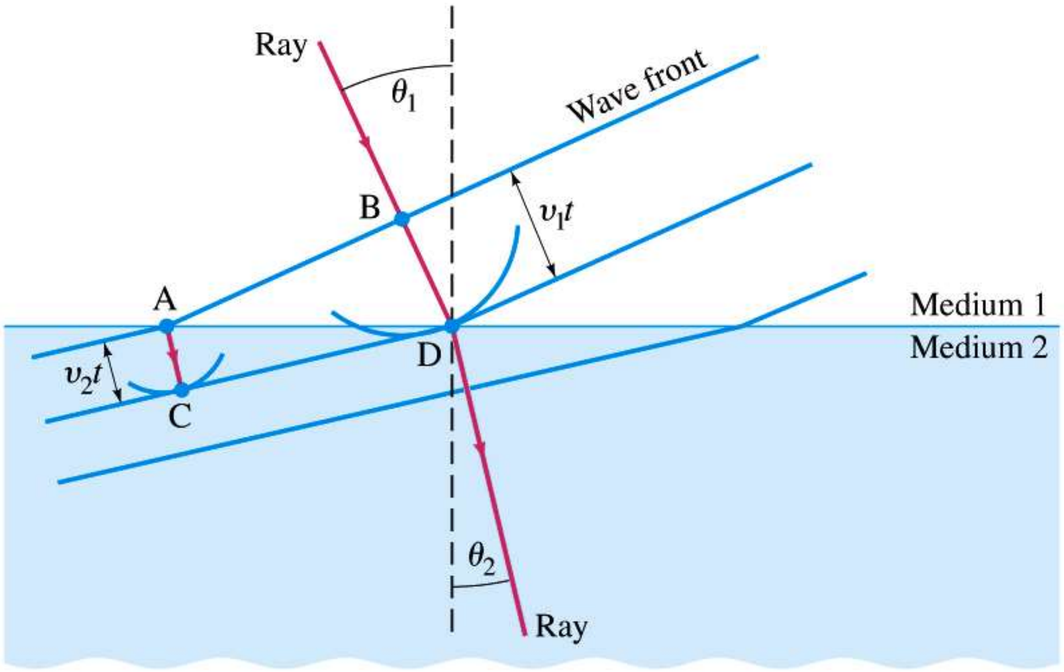


Diffraction



- Diffraction – Bending of light into the shadow region
- Grimaldi - 17th Century observation of diffraction
- Diffraction vs. Refraction?

Explanation of Snell's Law



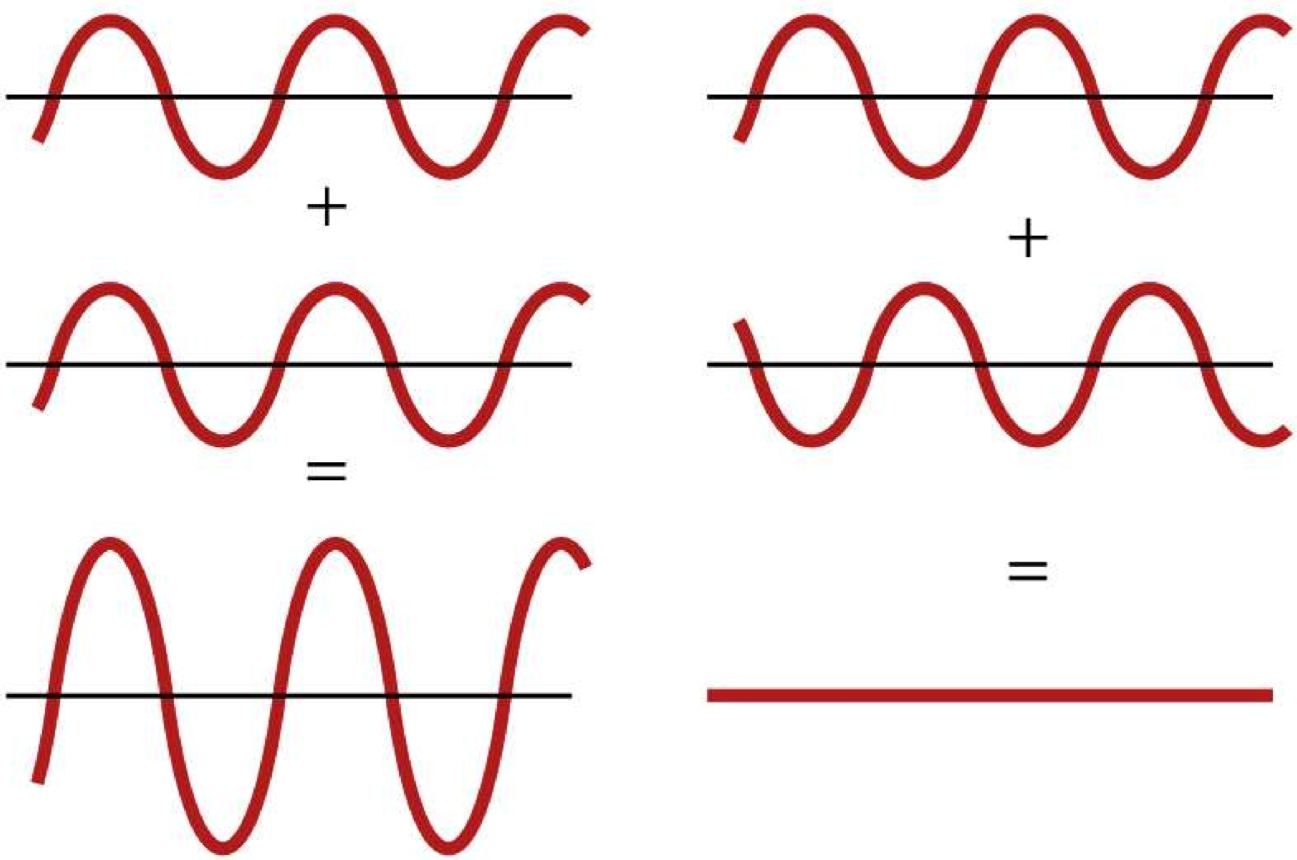
$$\sin\theta_1 = \frac{BD}{AD} = \frac{v_1 \Delta t}{AD}$$

$$\sin\theta_2 = \frac{AC}{AD} = \frac{v_2 \Delta t}{AD}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1 \Delta t}{v_2 \Delta t} = \frac{c/n_1}{c/n_2}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Superposition of waves



(a)

(b)

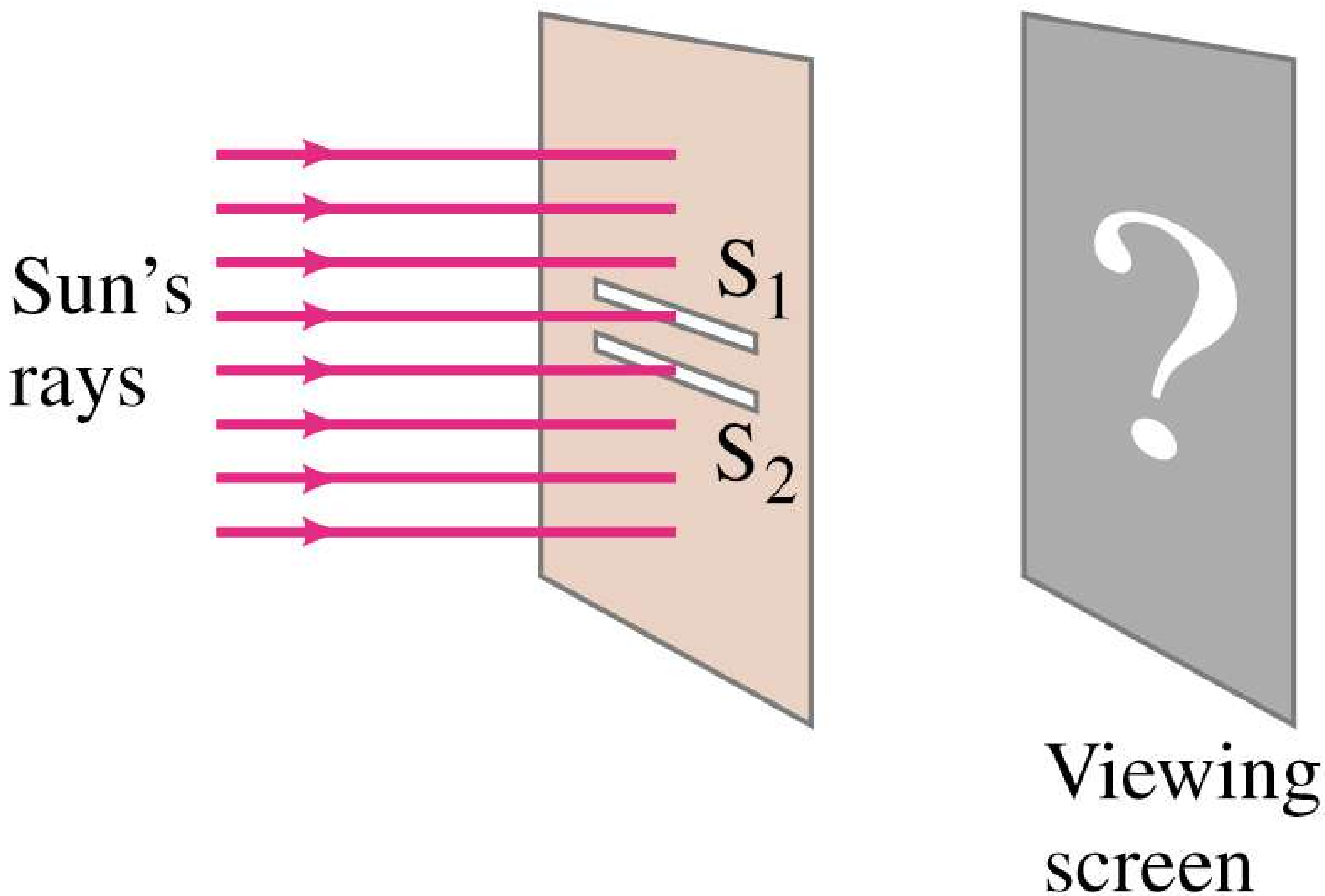
Interference

Interferen
Destructivece

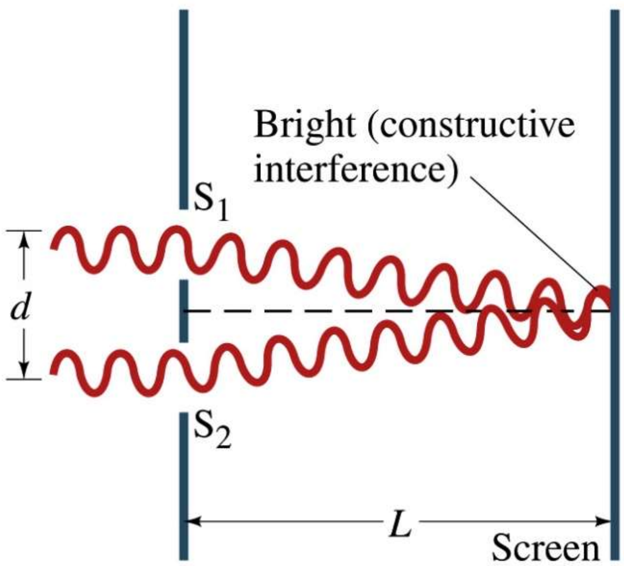
Conditions for Interference

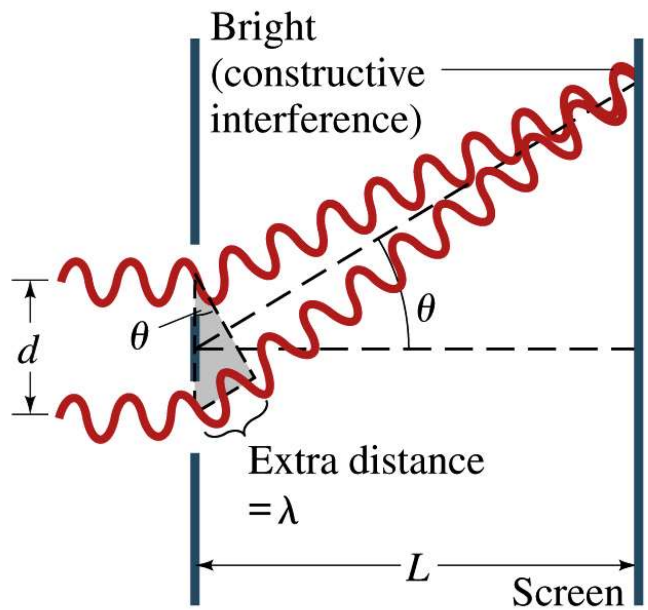
- To observe interference in light waves, the following two conditions must be met:
  1) The sources must be **coherent**
     - They must maintain a constant phase with respect to each other
  2) The sources should be **monochromatic**
     - Monochromatic means they have a single wavelength

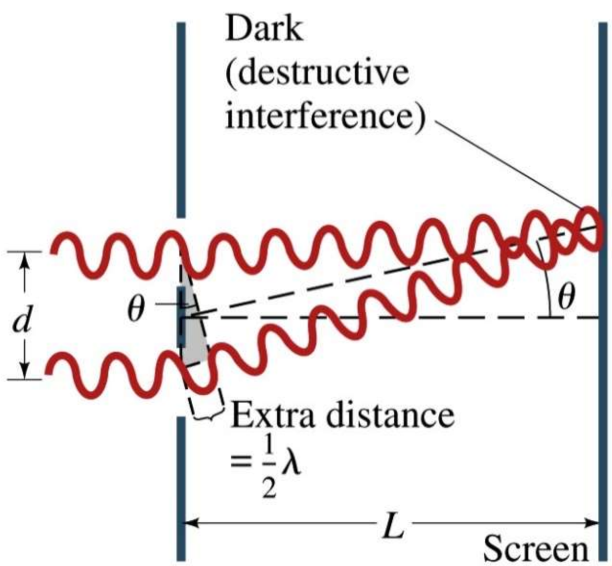
Young's Experiment



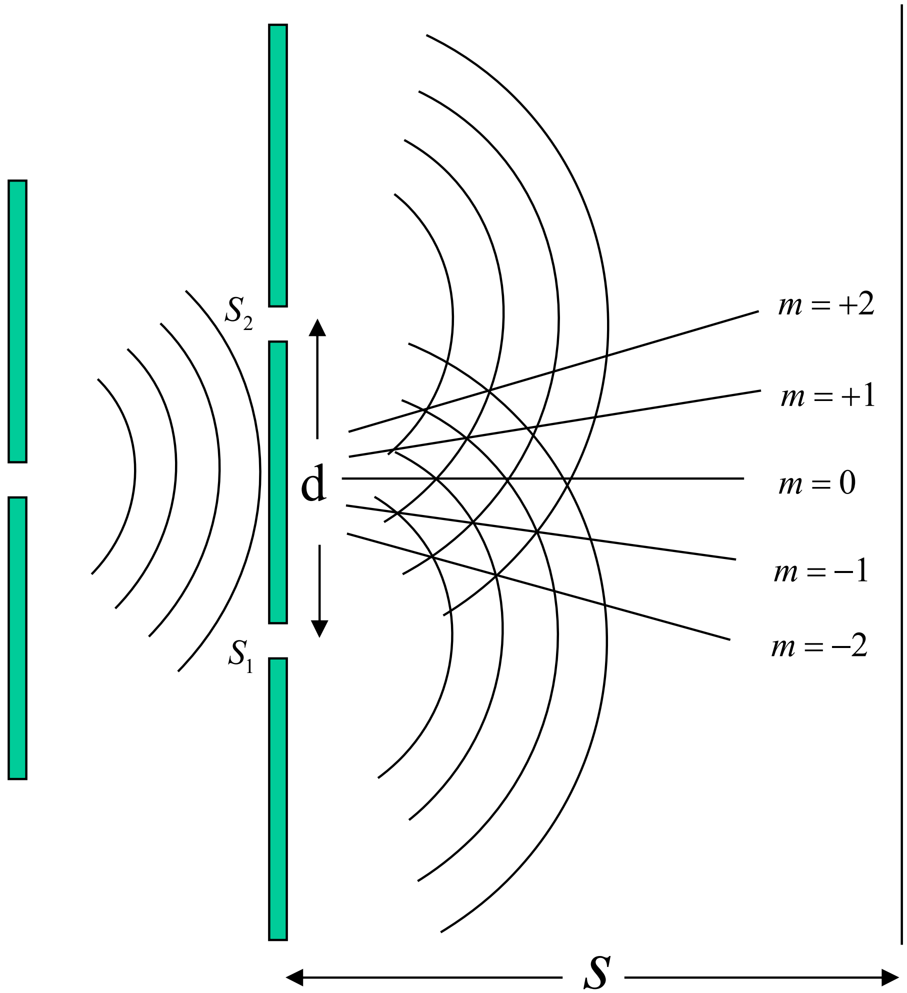
Young's Experiment







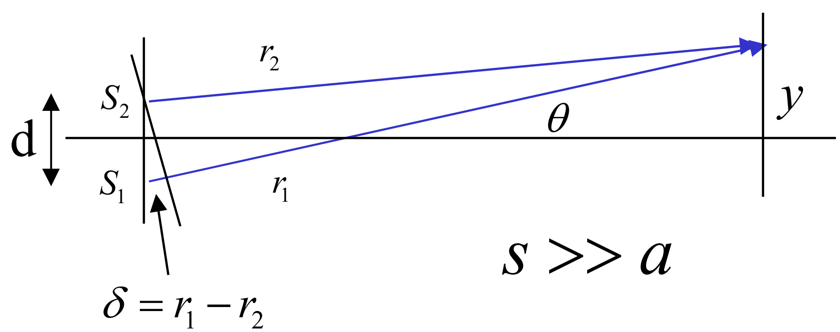
Young's Experiment



$$\delta = r_1 - r_2 = m\lambda$$

$$\sin\theta \approx \frac{\delta}{d}$$

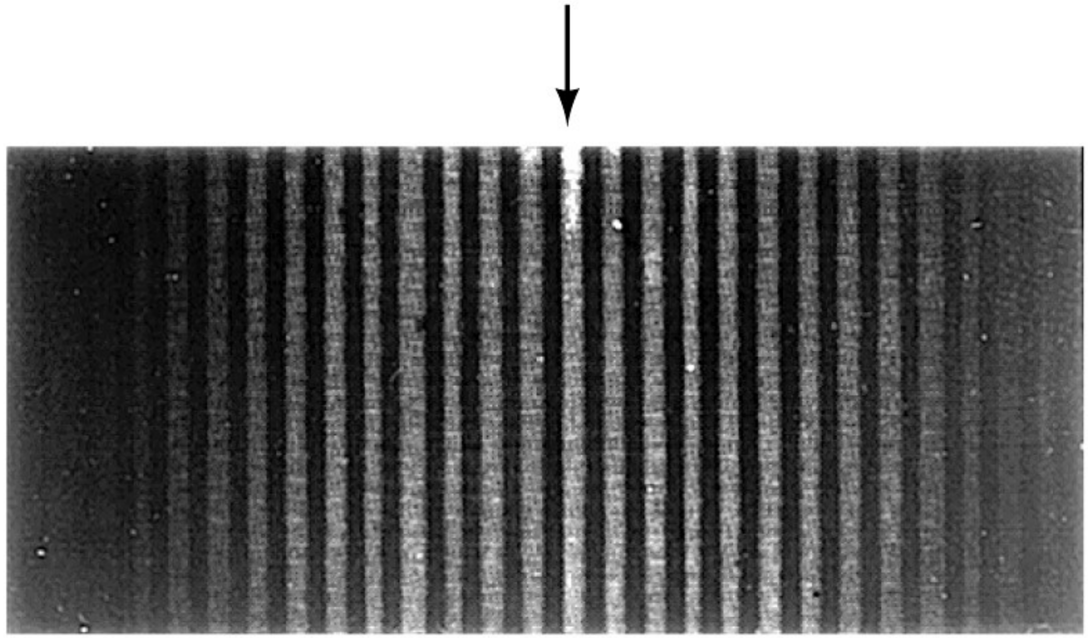
$$\delta \approx d\sin\theta = m\lambda$$

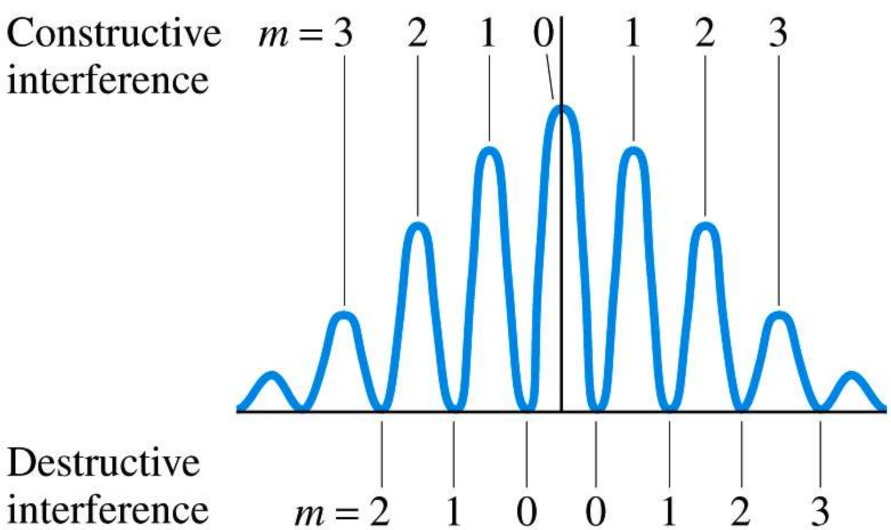


$$\delta = r_1 - r_2 = \left(m + \frac{1}{2}\right)\lambda$$

$$\delta \approx d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

What the pattern looks like

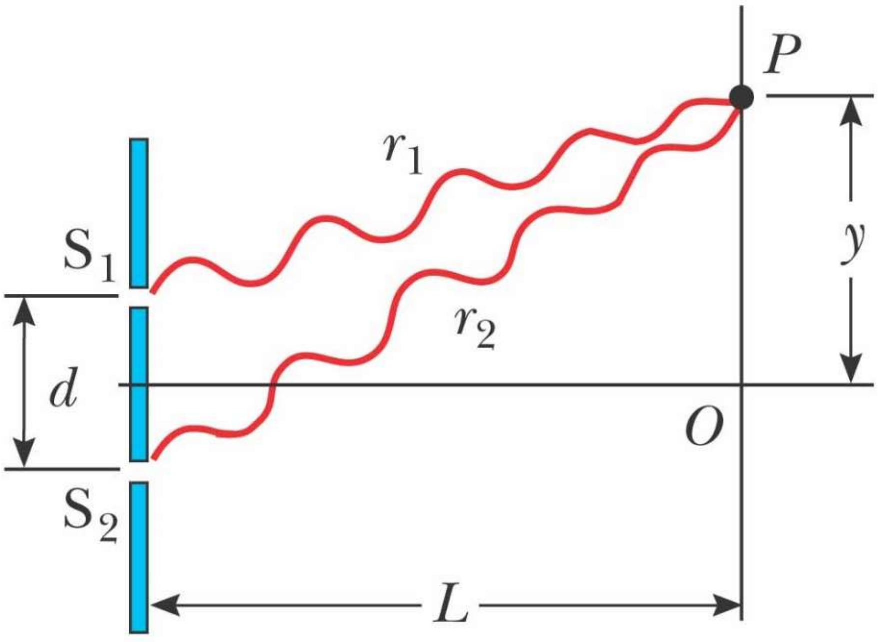




Intensity Distribution, Electric Fields

- The magnitude of each wave at point *P* can be found
  - $E_1 = E_o \sin \omega t$
  - $E_2 = E_o \sin (\omega t + \varphi)$
  - Both waves have the same amplitude, E_o

$$\varphi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d \sin\theta$$



Intensity Distribution, Resultant Field

- The magnitude of the resultant electric field comes from the superposition principle
  - $E_P = E_1 + E_2 = E_o[\sin \omega t + \sin(\omega t + \varphi)]$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

- This can also be expressed as

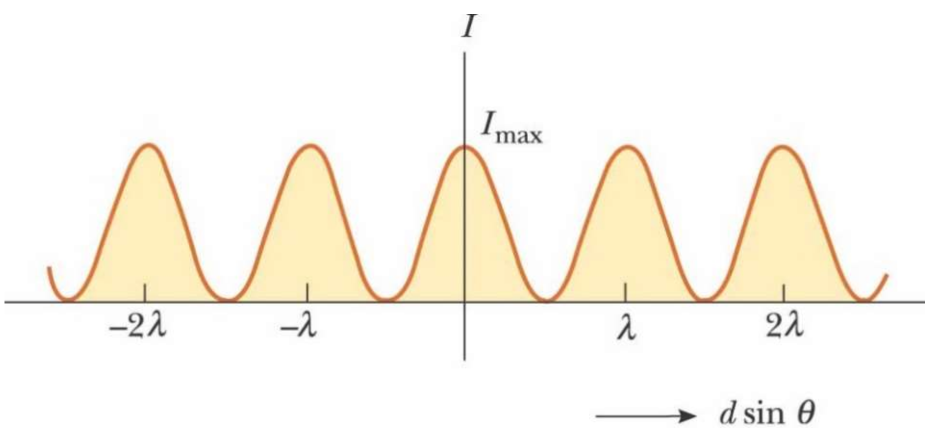
$$E_P = 2E_o \cos\left(\frac{\varphi}{2}\right)\sin\left(\omega t + \frac{\varphi}{2}\right)$$

  - E_P has the same frequency as the light at the slits
  - The magnitude of the field is multiplied by the factor 2 cos (φ / 2)

Intensity Distribution, Equation

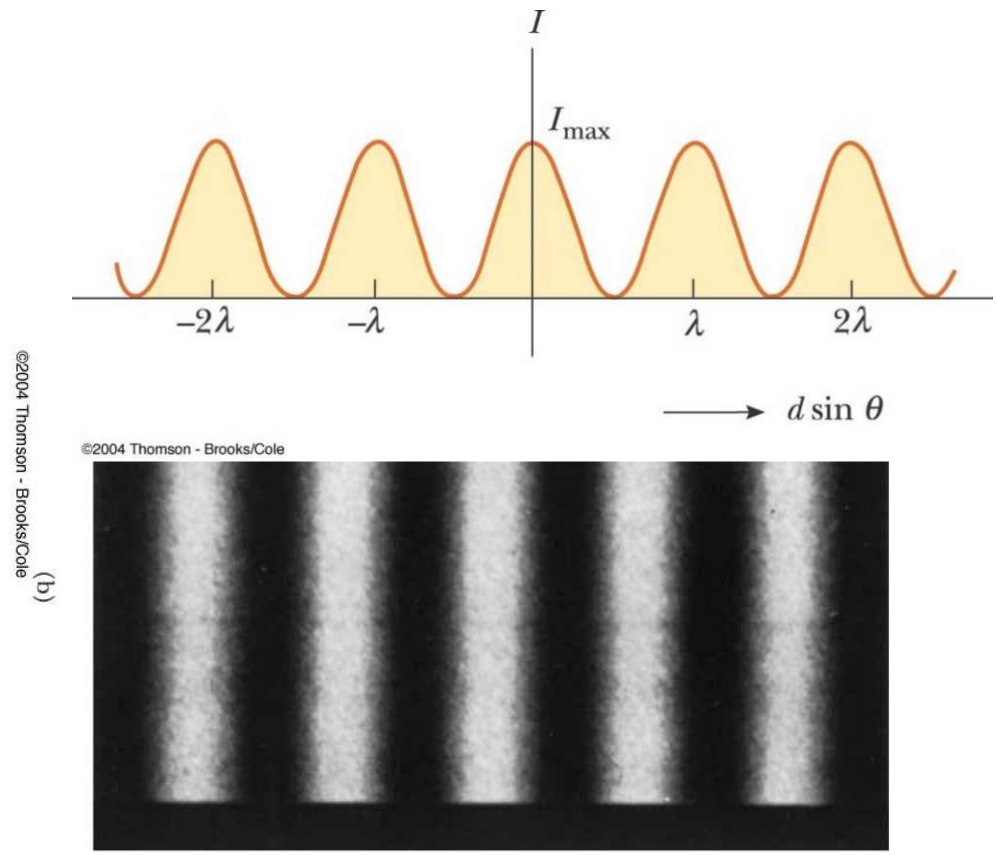
- The expression for the intensity comes from the fact that *the intensity of a wave is proportional to the square of the resultant electric field magnitude at that point*
- The intensity therefore is

$$I = I_{\text{max}} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \approx I_{\text{max}} \cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

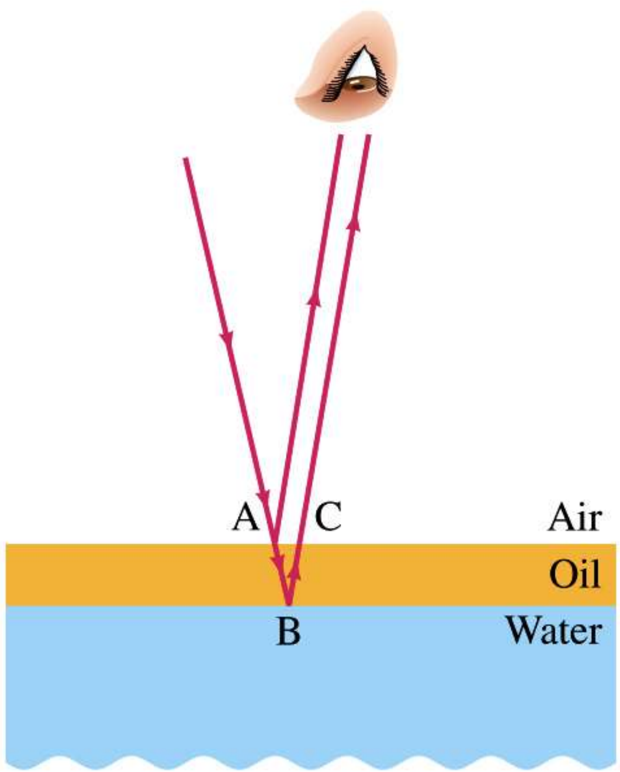


Resulting Interference Pattern

- The light from the two slits forms a visible pattern on a screen
- The pattern consists of a series of bright and dark parallel bands called *fringes*
- *Constructive interference* occurs where a bright fringe occurs
- *Destructive interference* results in a dark fringe



Thin Films



Constructive Interference (maxima)

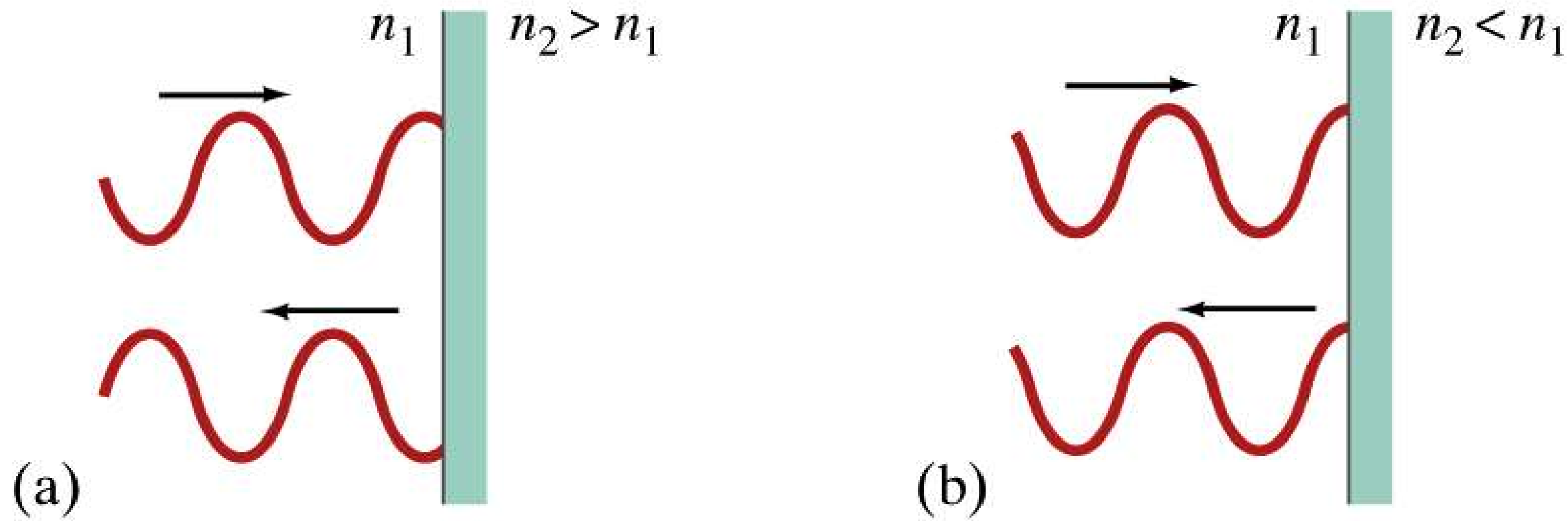
$$d_{ABC} = m\lambda_n$$

$$\lambda_n = \frac{\lambda_o}{n}$$

Destructive Interference (minima)

$$d_{ABC} = \left(m + \frac{1}{2}\right)\lambda_n$$

Phase shift on reflection

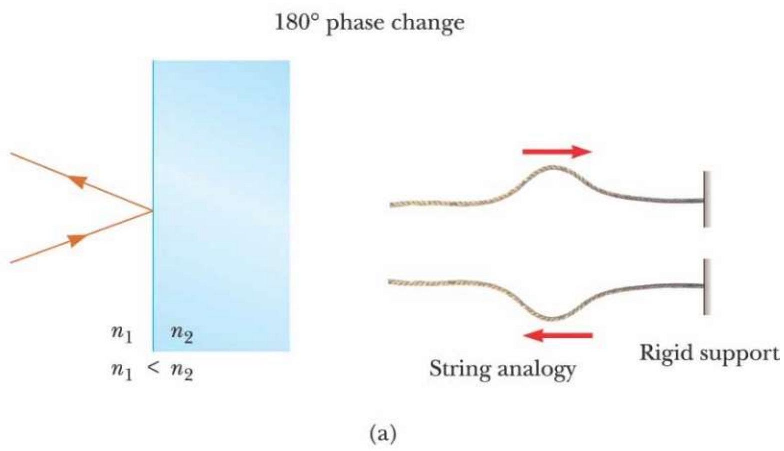


External Reflection   Reflection

Now, If one reflection is internal and one reflection is external half wavelength path differences will result in constructive interference

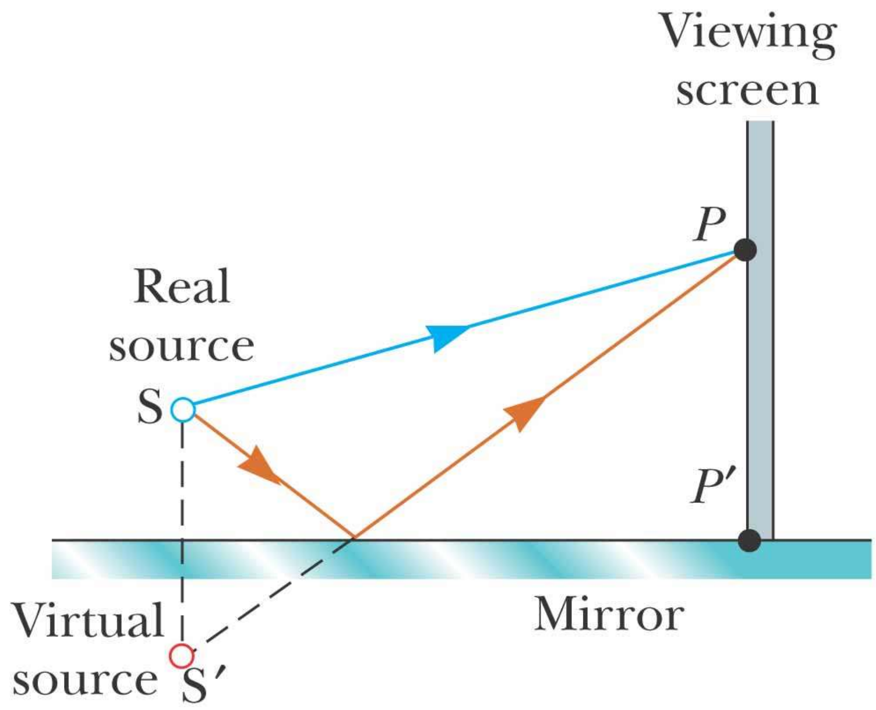
Phase Changes Due To Reflection

- An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling
  - Analogous to a pulse on a string reflected from a rigid support

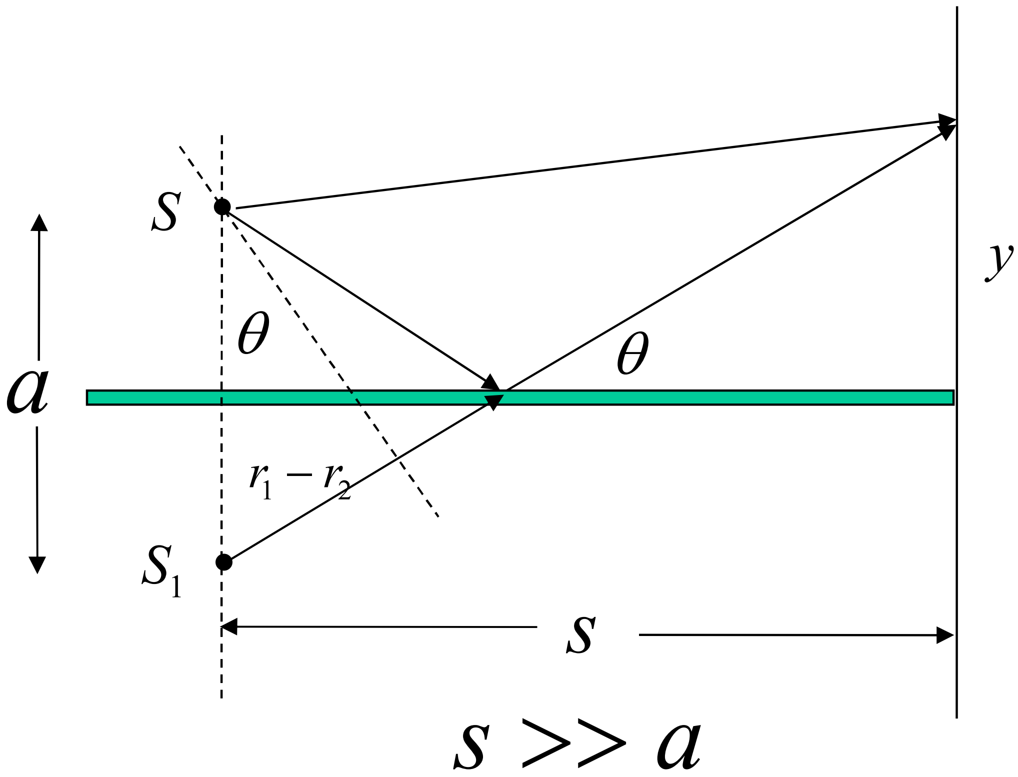


Lloyd's Mirror

- An arrangement for producing an interference pattern with a single light source
- Waves reach point *P* either by a direct path or by reflection
- The reflected ray can be treated as a ray from the source S' behind the mirror



Lloyd's Mirror



$$\frac{2\pi}{\lambda}\left(a\sin\theta_m\right) \pm \pi = m2\pi$$

Phase shift on reflection = π

$$a\sin\theta_m = \frac{\lambda}{2}\left(2m \mp 1\right) = \lambda\left(m \mp \frac{1}{2}\right) \approx a\theta_m \approx a\frac{y_m}{s}$$

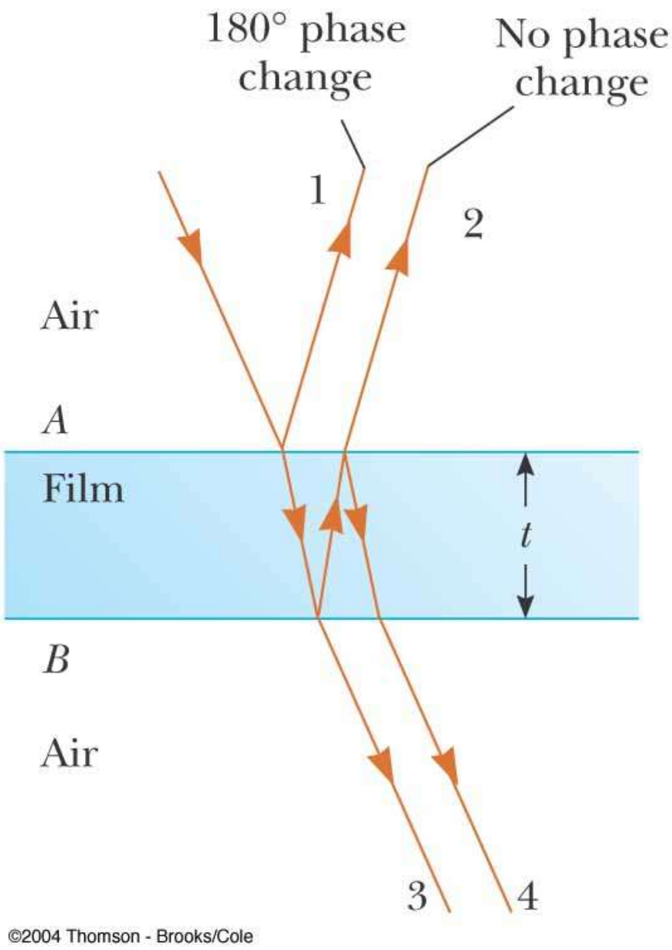
Interference in Thin Films Again

- Assume the light rays are traveling in air nearly normal to the two surfaces of the film
- Ray 1 undergoes a phase change of 180° with respect to the incident ray
- Ray 2, which is reflected from the lower surface, undergoes no phase change with respect to the incident wave
- For constructive interference

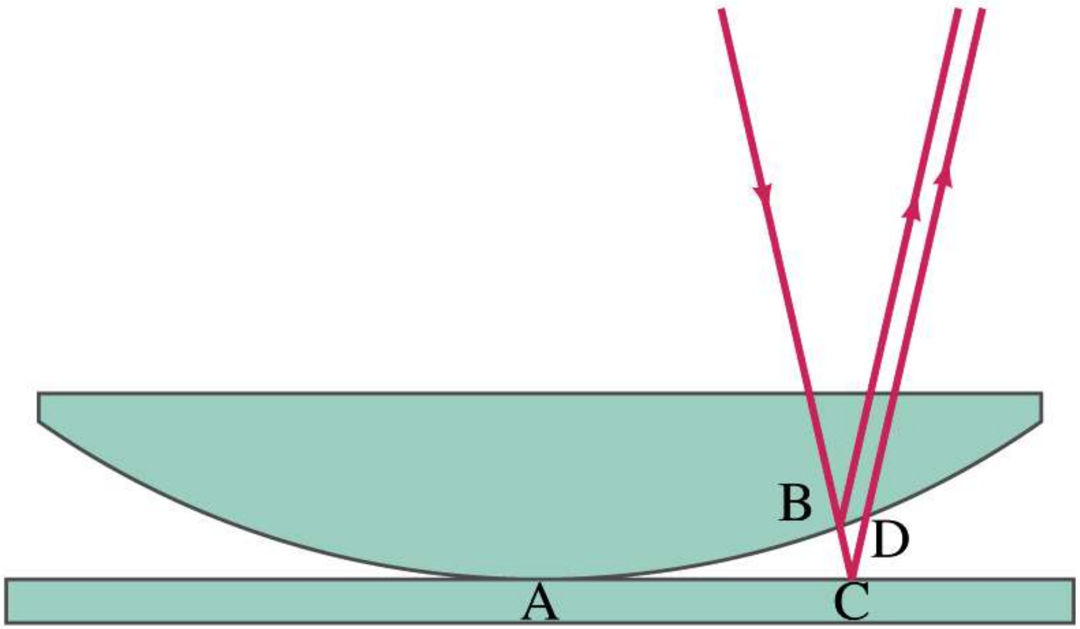
$$\delta = 2t = (m + \tfrac{1}{2})\lambda_n \quad (m = 0, 1, 2 \ldots)$$

- This takes into account both the difference in optical path length for the two rays and the 180° phase change
- For destructive interference

$$\delta = 2t = m\lambda_n \quad (m = 0, 1, 2 \ldots)$$



Newton's Rings



$$\lambda = \frac{\lambda_o}{n}$$

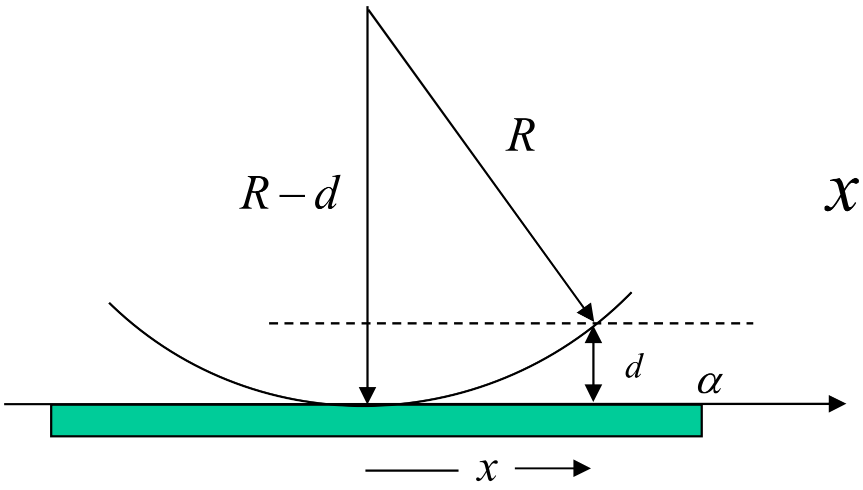
Max - bright

$$2d_m = \lambda\left(m + \frac{1}{2}\right)$$

Min - dark

$$2d_m = m\lambda$$

Newton's Rings



$$R^2 = x^2 + (R-d)^2$$

$$x^2 = R^2 - (R-d)^2 = R^2 - R^2 - d^2 + 2Rd$$

$$x^2 = -d^2 + 2Rd \approx 2Rd$$

Min $\quad 2d = m\lambda_n$

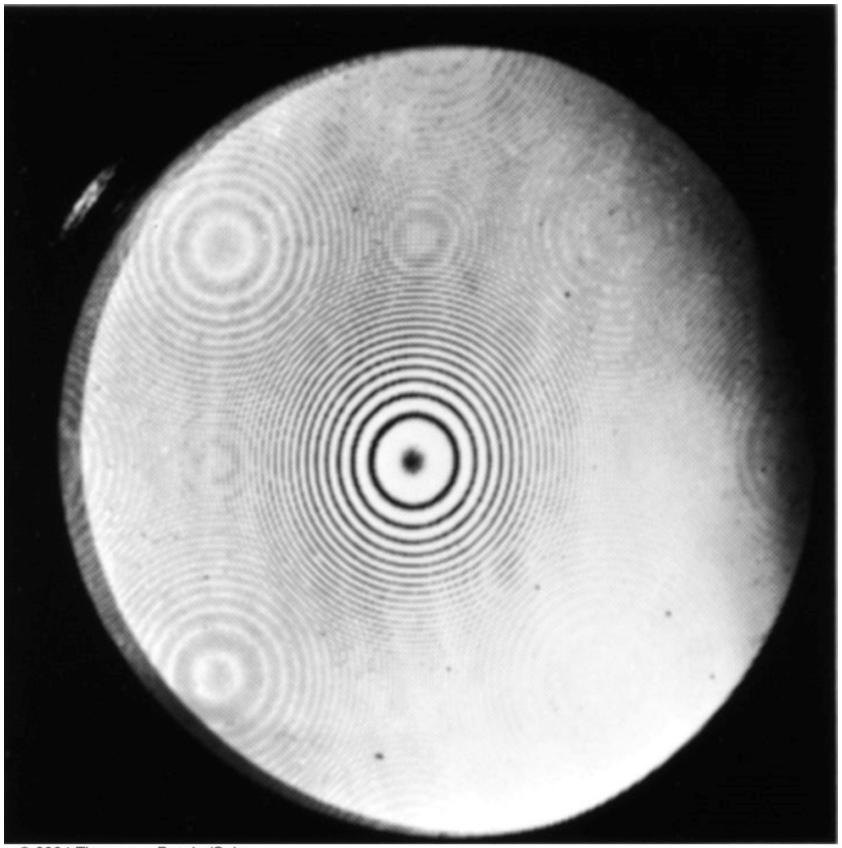
$$2\frac{x_m^2}{2R} = m\frac{\lambda_o}{n}$$

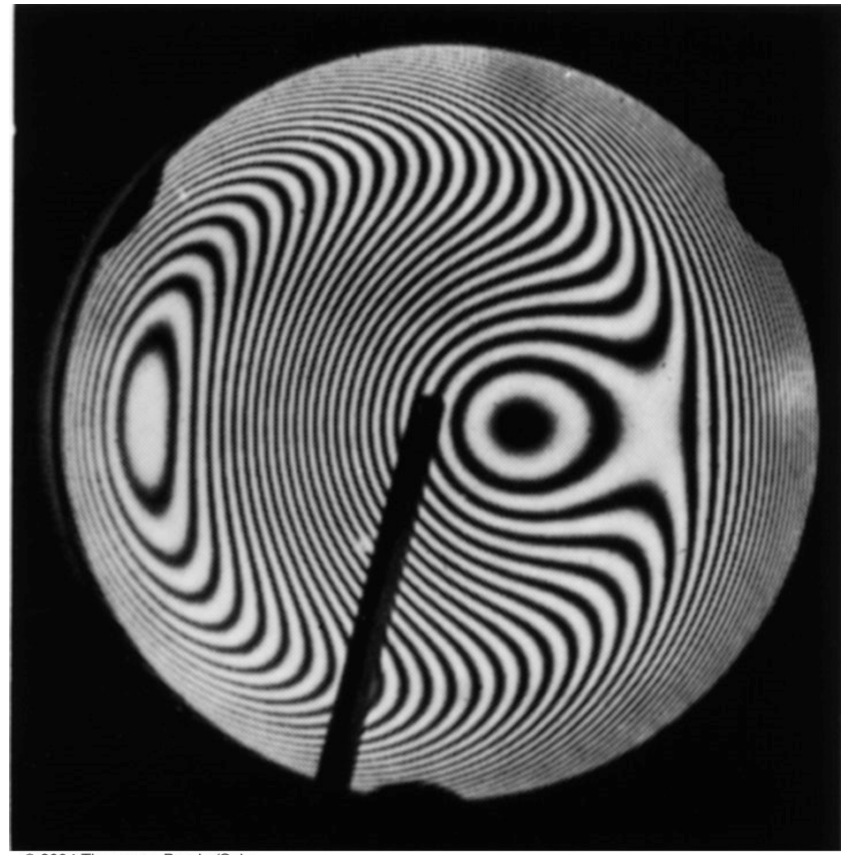
$$x_{min} = \sqrt{\frac{\lambda_o R m}{n}}$$

Newtons rings are a special case of Fizeau fringes. They are useful for testing surface accuracy of a lens.

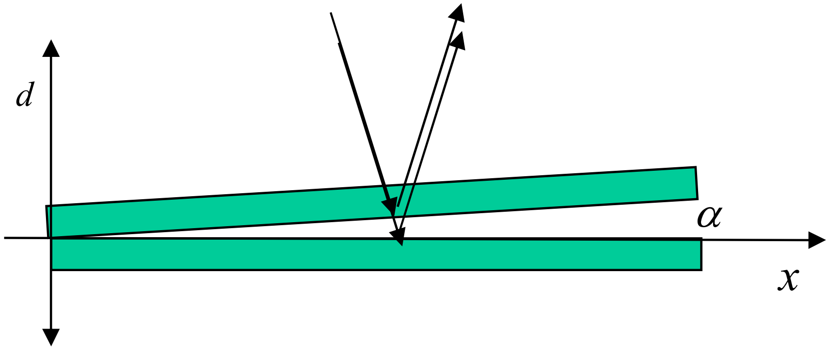
$$\lambda = \frac{\lambda_o}{n}$$

Max $\quad x_{max} = \sqrt{\frac{\lambda_o R}{n}\left(m + \frac{1}{2}\right)}$





Wedges – Fringes of Equal Thickness



$$t = x\alpha$$

Min (destructive)

$$2t = m\lambda = m\frac{\lambda_o}{n}$$

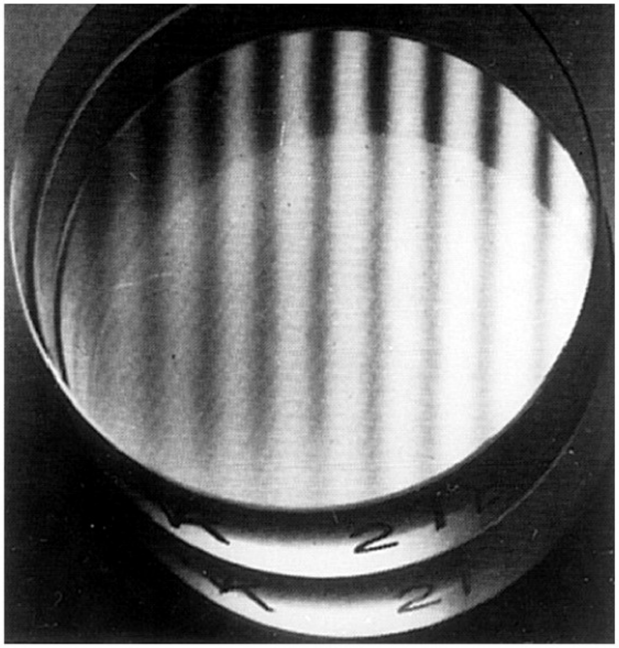
Max (constructive)

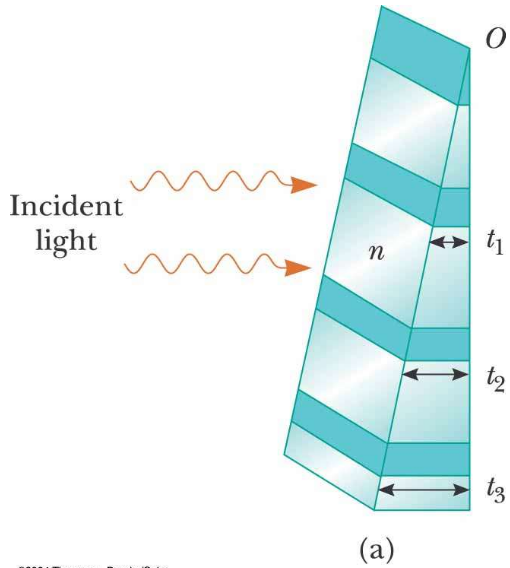
$$2t = \lambda\left(m + \frac{1}{2}\right)$$

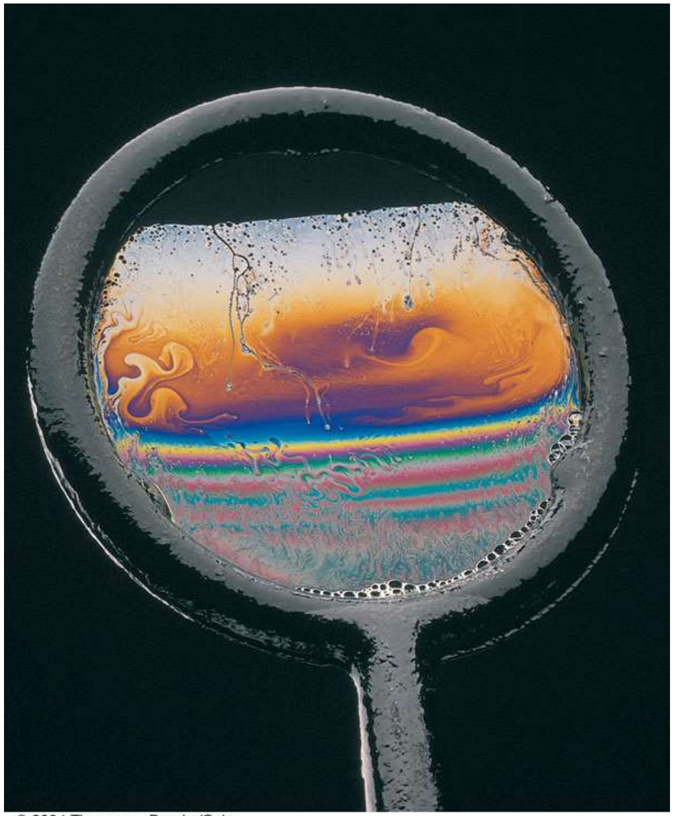
$$x_m == \frac{\lambda}{2\alpha}\left(m + \frac{1}{2}\right)$$

$$\Delta x_m = \frac{\lambda}{2\alpha}$$

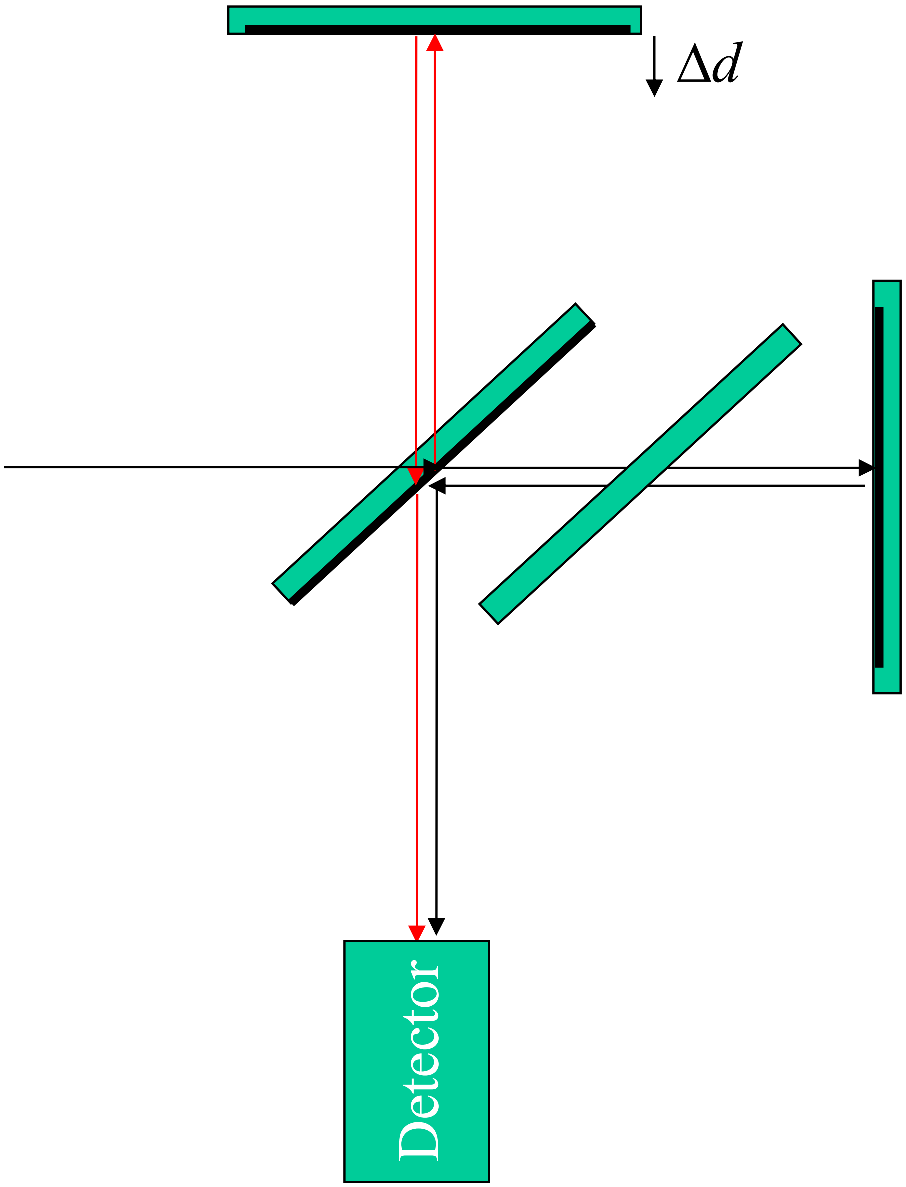
Example: thin film of air







Michelson Interferometer



- A lens can be used to form fringes of equal inclination (rings)
- Tilting the mirrors can cause fringes of equal thickness.
- Accurate length measurements are accomplished by fringe counting as one of the mirrors is moved.

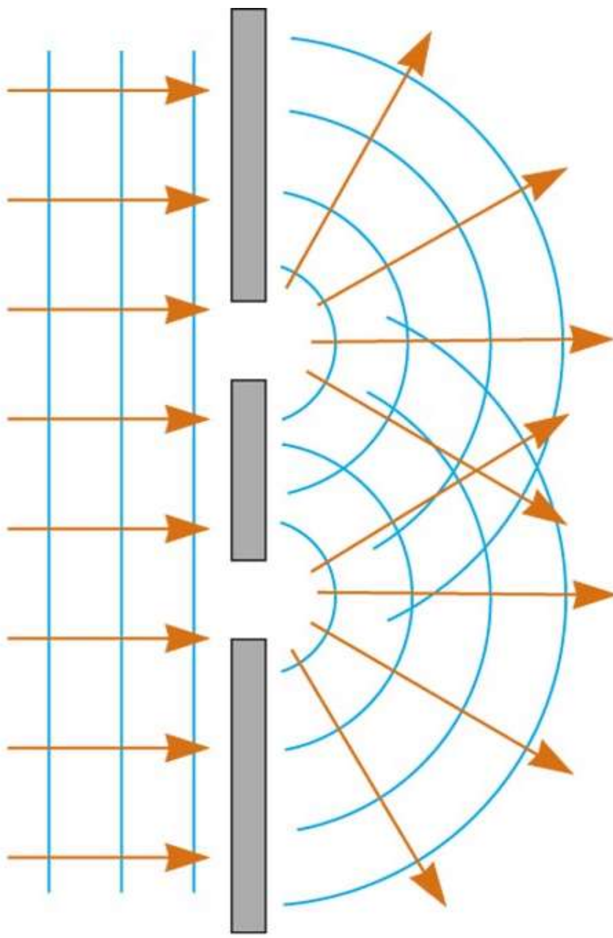
$$\delta = 2\Delta d = m\lambda$$

$$\Delta d = \frac{m\lambda}{2}$$

3.4. DIFFRACTION PATTERNS AND POLARIZATION

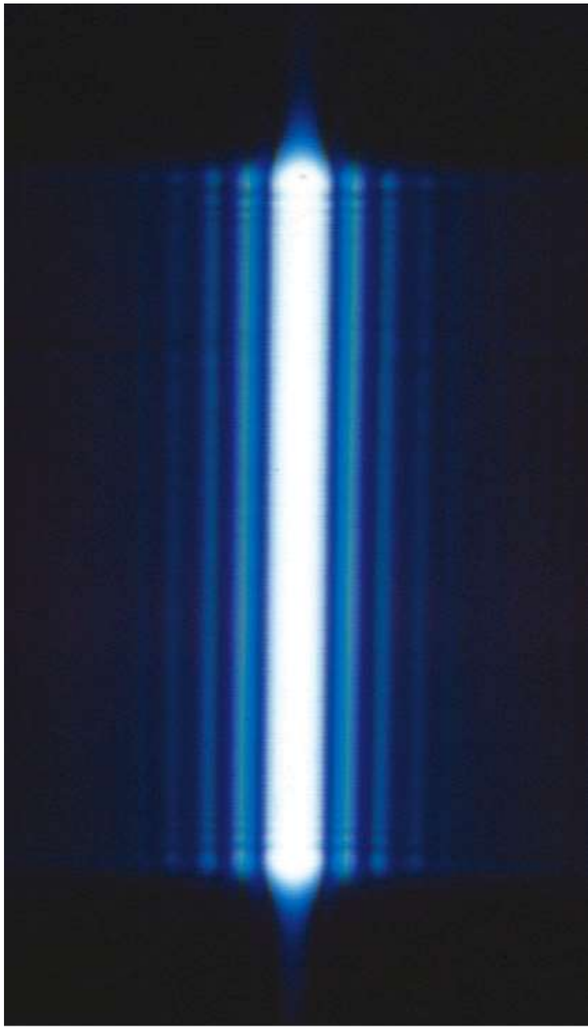
Diffraction

- Huygen's principle requires that the waves spread out after they pass through slits
- This spreading out of light from its initial line of travel is called diffraction
- In general, diffraction occurs when waves pass through small openings, around obstacles or by sharp edges



Diffraction

- A single slit placed between a distant light source and a screen produces a diffraction pattern
- It has a broad, intense central band
- The central band is flanked by a series of narrower, less intense secondary bands called secondary maxima
- The central band will also be flanked by a series of dark bands called minima
- This result cannot be explained by geometric optics

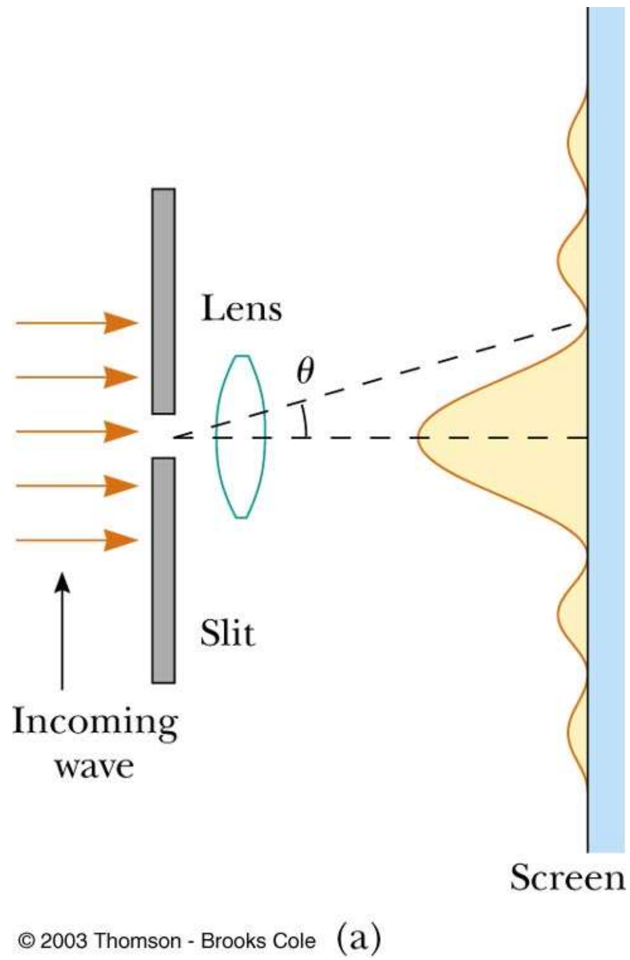


Fraunhofer Diffraction

- Fraunhofer Diffraction occurs when the rays leave the diffracting object in parallel directions

- The screen is very far from the slit and the lens is converging

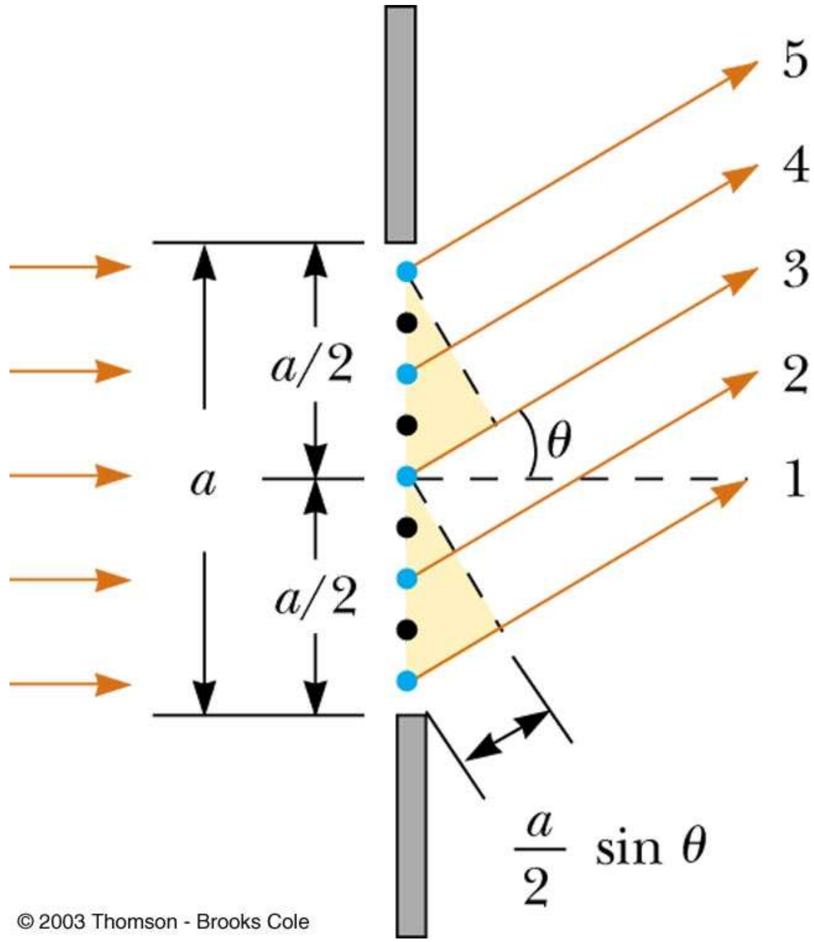
- A bright fringe is seen along the axis ($\theta = 0$) with alternating bright and dark fringes on each side



Joseph von Fraunhofer

Single Slit Diffraction

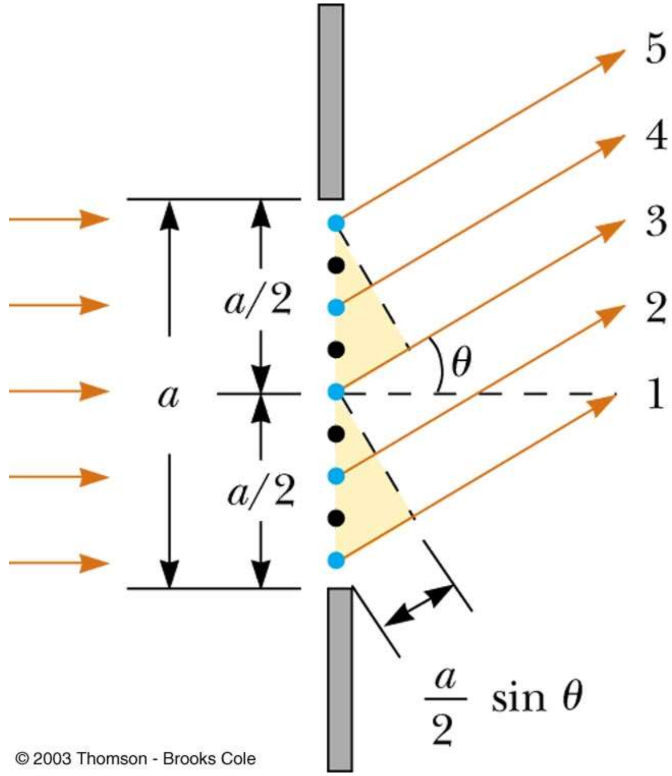
- According to Huygen's principle, each portion of the slit acts as a source of waves
- The light from one portion of the slit can interfere with light from another portion
- The resultant intensity on the screen depends on the direction θ
- All the waves that originate at the slit are in phase



Single Slit Diffraction

- Wave 1 travels farther than wave 3 by an amount equal to the path difference $(a / 2) \sin \theta$
- If this path difference is exactly half of a wavelength, the two waves cancel each other and destructive interference results
- In general, destructive interference occurs for a single slit of width a when

$$\sin \theta_{dark} = m\lambda / a; \;\; m = \pm 1, \pm 2, \ldots$$

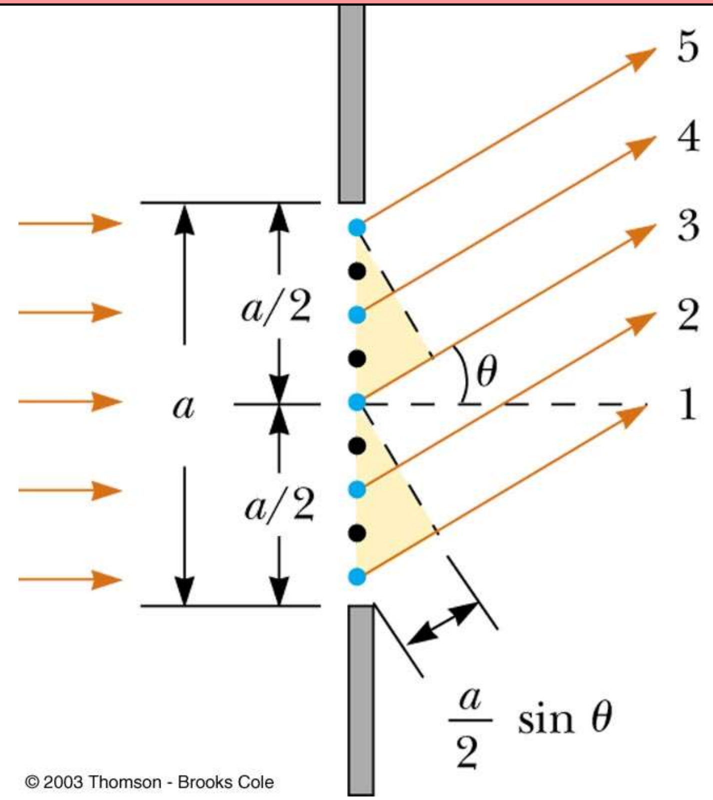


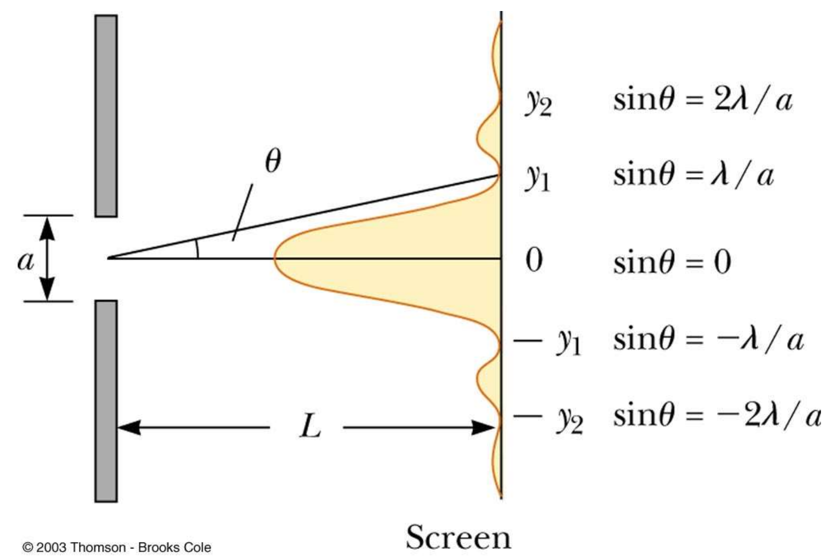
$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \qquad \sin\theta = \frac{\lambda}{a}$$

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2} \qquad \sin\theta = \frac{2\lambda}{a}$$

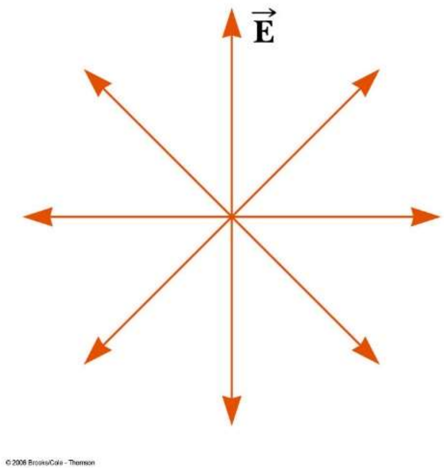
Single Slit Diffraction

- The general features of the intensity distribution are shown
- A broad central bright fringe is flanked by much weaker bright fringes alternating with dark fringes
- The points of constructive interference lie approximately halfway between the dark fringes





Polarization of Light



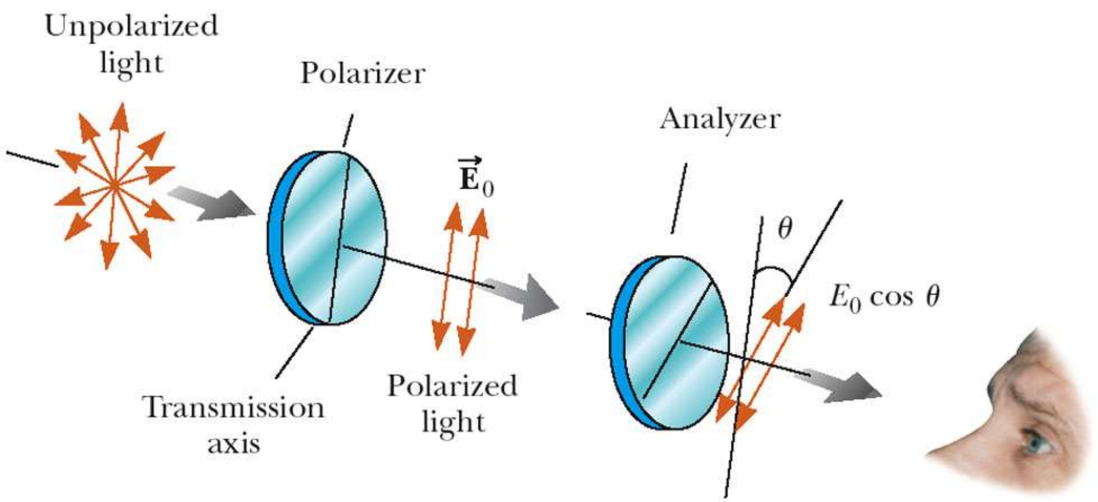
- An unpolarized wave: each atom produces a wave with its own orientation of $\vec{E}$, so all directions of the electric field vector are equally possible and lie in a plane perpendicular to the direction of propagation

- A wave is said to be linearly polarized if the resultant electric field vibrates in the same direction at all times at a particular point

- Polarization can be obtained from an unpolarized beam by selective absorption, reflection, or scattering

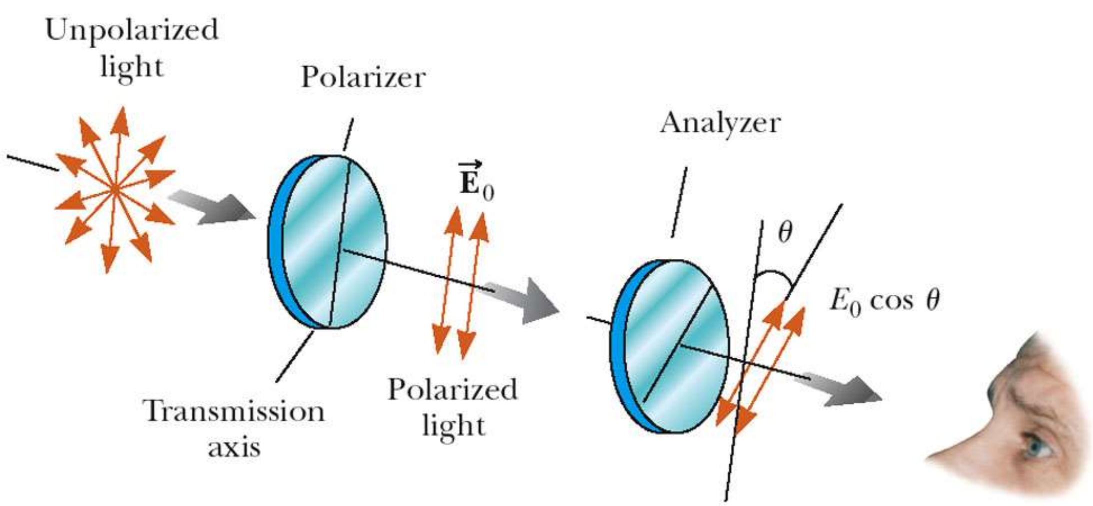
Polarization by Selective Absorption

- The most common technique for polarizing light
- Uses a material that transmits waves whose electric field vectors in the plane are parallel to a certain direction and absorbs waves whose electric field vectors are perpendicular to that direction



Polarization by Selective Absorption

- The intensity of the polarized beam transmitted through the second polarizing sheet (the analyzer) varies as $I = I_o \cos^2 \theta$, where I_o is the intensity of the polarized wave incident on the analyzer
- This is known as Malus' Law and applies to any two polarizing materials whose transmission axes are at an angle of θ to each other



Étienne-Louis Malus
1775 – 1812

Polarization by Reflection

- When an unpolarized light beam is reflected from a surface, the reflected light can be completely polarized, partially polarized, or unpolarized
- It depends on the angle of incidence
- If the angle is 0° or 90°, the reflected beam is unpolarized
- For angles between this, there is some degree of polarization
- For one particular angle, the beam is completely polarized

Polarization by Reflection

- The angle of incidence for which the reflected beam is completely polarized is called the polarizing (or Brewster's) angle, θ_p

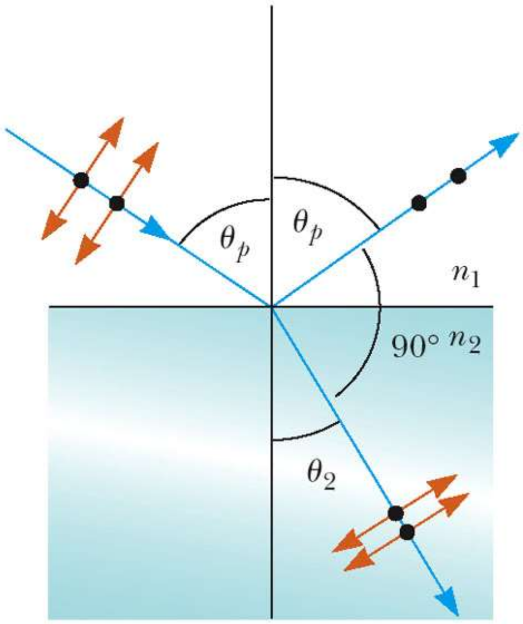
- Brewster's Law relates the polarizing angle to the index of refraction for the material

Sir David Brewster

$$\theta_p + 90° + \theta_2 = 180° \qquad \theta_2 = 90° - \theta_p$$

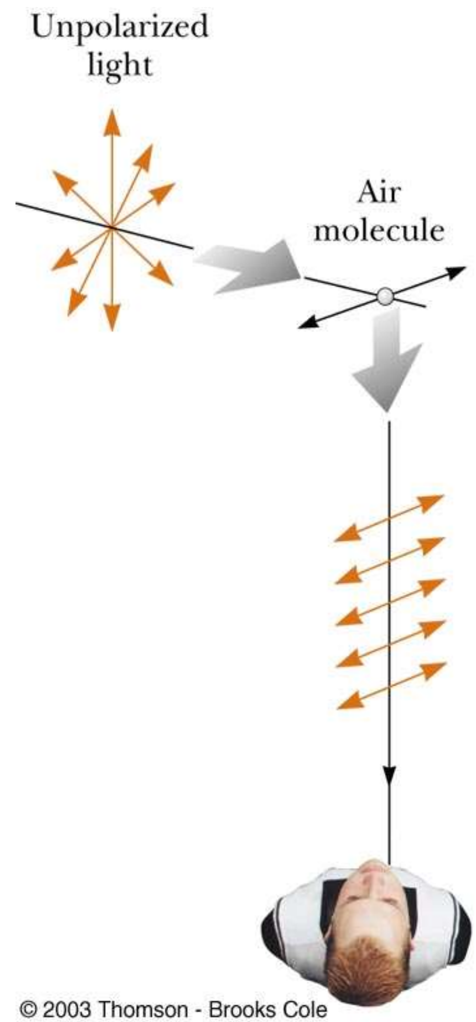
$$n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\theta_2} = \frac{\sin\theta_p}{\cos\theta_p}$$

$$n = \frac{\sin\theta_p}{\cos\theta_p} = \tan\theta_p$$



Polarization by Scattering

- When light is incident on a system of particles, the electrons in the medium can scatter – absorb and reradiate – part of the light (e.g., sunlight reaching an observer on the ground becomes polarized)
- The horizontal part of the electric field vector in the incident wave causes the charges to vibrate horizontally
- The vertical part of the vector simultaneously causes them to vibrate vertically
- Horizontally and vertically polarized waves are emitted



Optical Activity

- Certain materials display the property of optical activity
- A substance is optically active if it rotates the plane of polarization of transmitted light
- Optical activity occurs in a material because of an asymmetry in the shape of its constituent materials