# Module 2:
# Electromagnetic field;
# Electromagnetic oscillations and waves

# 1.Maxwell’s Equations

# 1.1. Equations for Electricity and Magnetism

- Gauss' law for electric fields

$$\int \vec{E} \cdot \overrightarrow{dA} = q / \varepsilon_0$$

the electric flux out of a volume = (charge inside)/$\varepsilon_0$.

- Gauss' law for magnetic fields

$$\int \vec{B} \cdot \overrightarrow{dA} = 0$$

- There is no such thing as magnetic charge: magnetic field lines just *circulate*, so for any volume they flow out of, they flow back into it somewhere else.

# 1.1. Equations for Electricity and Magnetism

- Electrostatics: (no changing fields)

$$\oint \vec{E} \cdot \overrightarrow{d\ell} = 0$$

around any closed curve: this means the work done against the electric field from A to B is independent of path, the field is conservative: a potential energy can be defined.

- Faraday's law of induction: in the presence of a changing magnetic field, the above equation becomes:

$$\oint \vec{E} \cdot \overrightarrow{d\ell} = -d/dt\left(\int \vec{B} \cdot \overrightarrow{dA}\right) = -d\Phi_B/dt$$

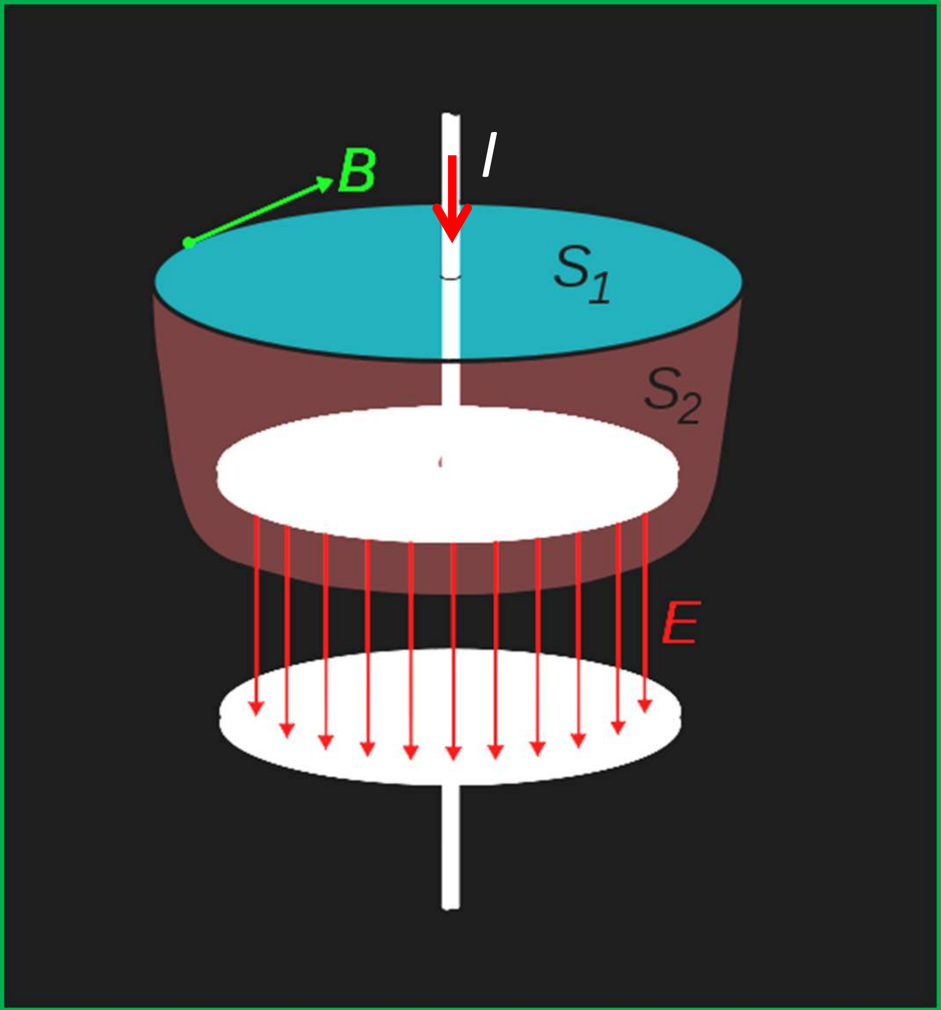
the integral is over an area "roofing" the path. A changing magnetic flux through the loop induces an emf.

# 1.1. Equations for Electricity and Magnetism

- Mangnetostatics:

$$\oint \vec{B} \cdot \overrightarrow{d\ell} = \mu_0 I$$

around any closed curve: $I$ is the total current flow across any surface roofing the closed curve of integration.



B
I
S1
S2
E

# 1.2. Maxwell's Equations

- The four equations that together give a complete description of electric and magnetic fields are known as Maxwell's equations:

$$\int \vec{E} \cdot \overrightarrow{dA} = q / \varepsilon_0 \qquad \int \vec{B} \cdot \overrightarrow{dA} = 0$$

$$\oint \vec{E} \cdot \overrightarrow{d\ell} = -d\Phi_B / dt$$

$$\oint \vec{B} \cdot \overrightarrow{d\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell himself called this term the "displacement current": it produces magnetic field like a current.

# 2. Electromagnetic Waves

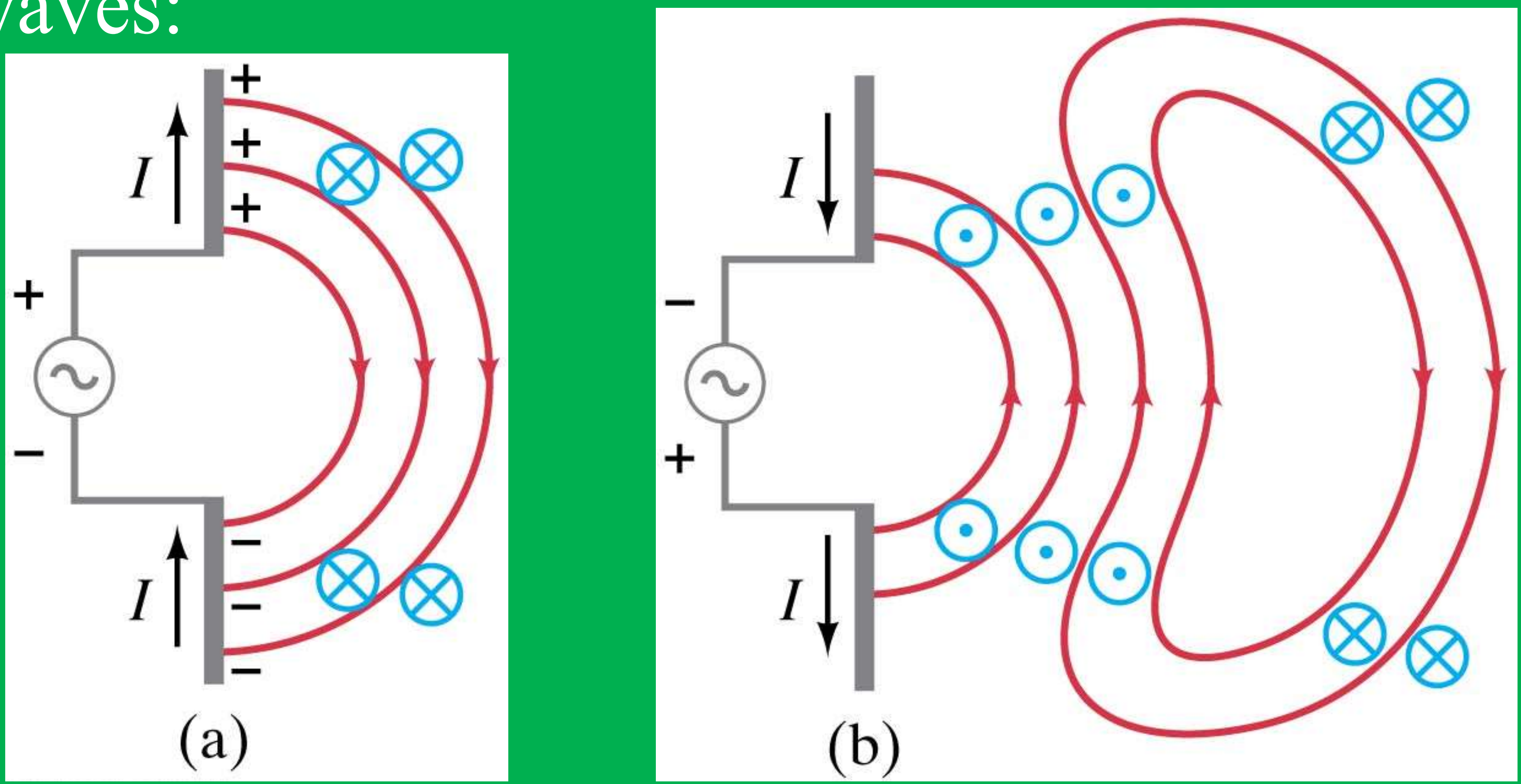
# 2.1. Production of Electromagnetic Waves

Since a changing electric field produces a magnetic field, and a changing magnetic field produces an electric field, once sinusoidal fields are created they can propagate on their own.

These propagating fields are called electromagnetic waves.

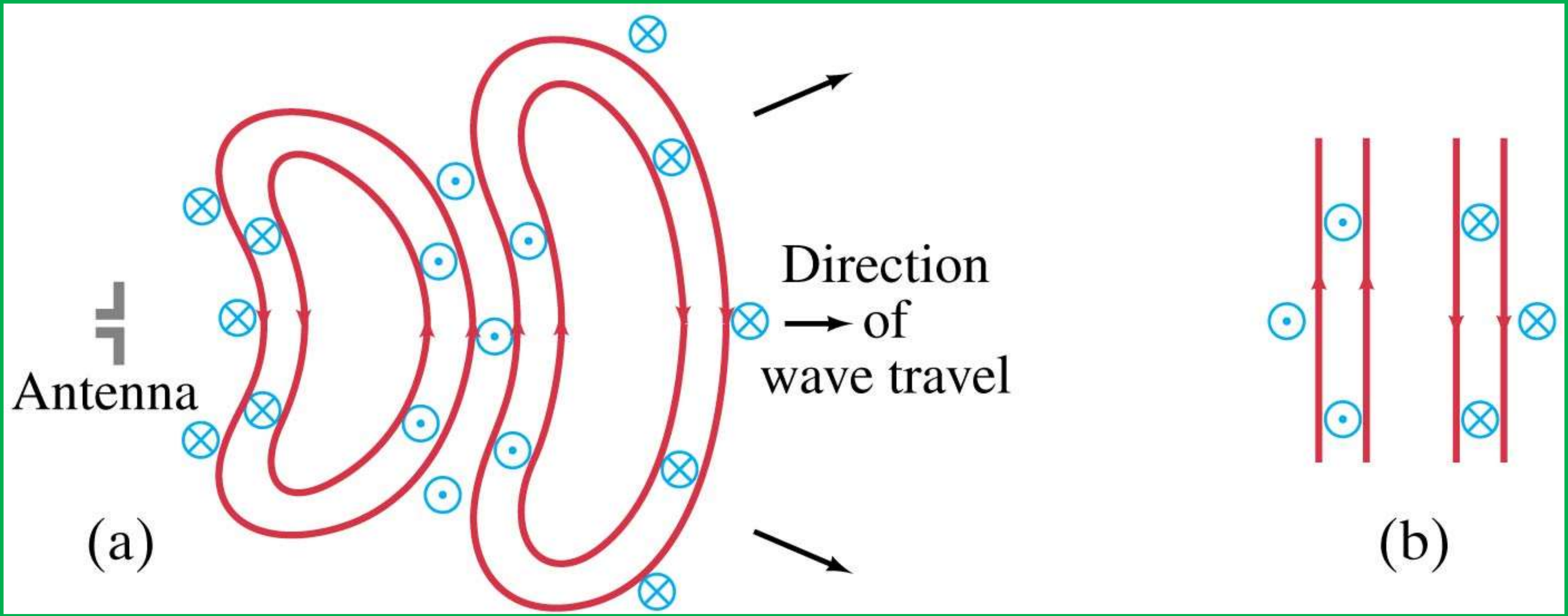
# 2.1. Production of Electromagnetic Waves

Oscillating charges will produce electromagnetic waves:



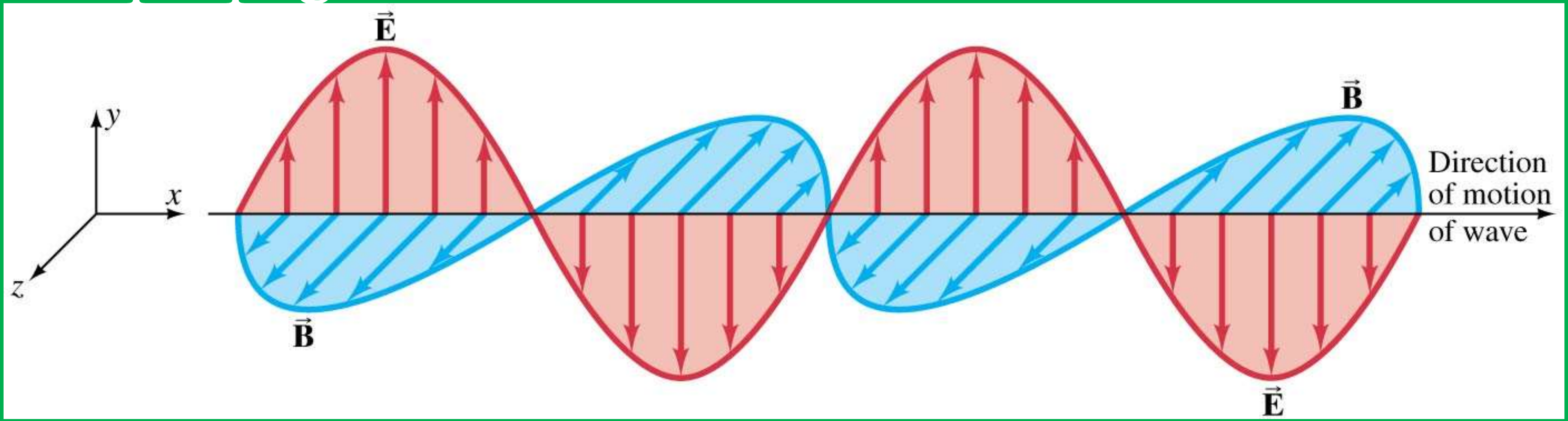
# 2.1. Production of Electromagnetic Waves

Far from the source, the waves are plane waves:



# 2.1. Production of Electromagnetic Waves

The electric and magnetic waves are perpendicular to each other, and to the direction of propagation.



# 2.1. Production of Electromagnetic Waves

When Maxwell calculated the speed of propagation of electromagnetic waves, he found:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}.$$

Using the known values of $\varepsilon_0$ and $\mu_0$ gives $c = 3.00 \times 10^8$ m/s.

This is the speed of light in a vacuum.

# 2.2. Light as an Electromagnetic Wave and the Electromagnetic Spectrum

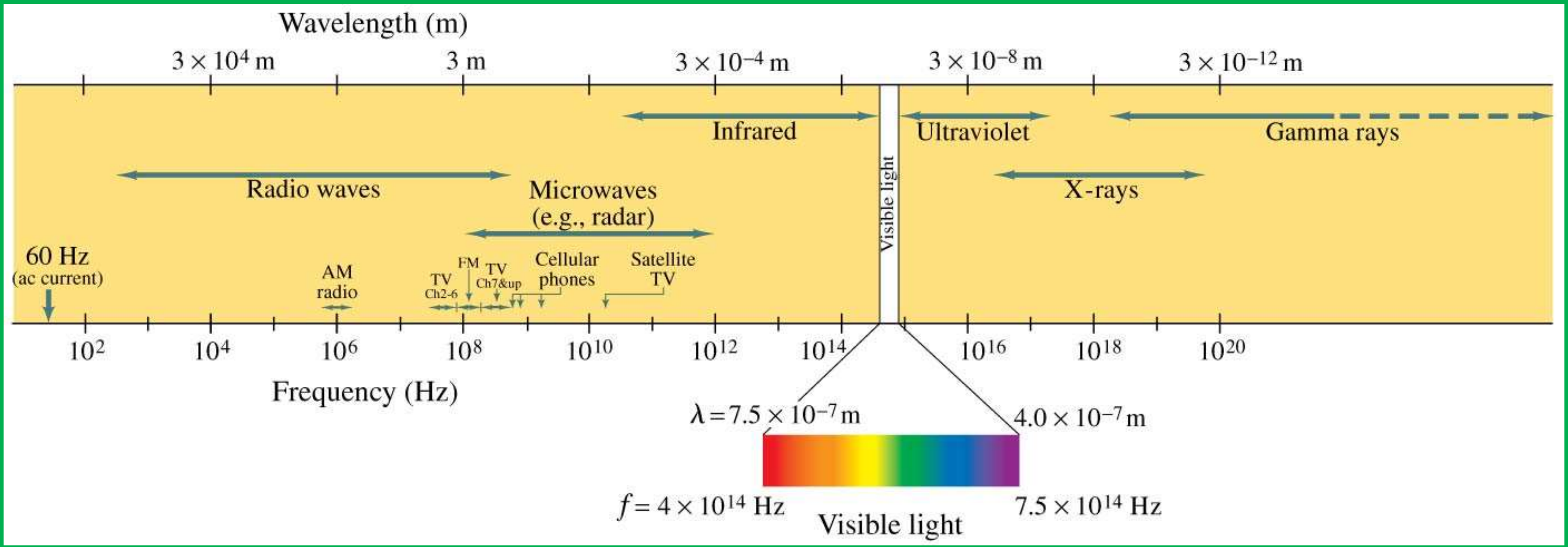
Light was known to be a wave. The production and measurement of electromagnetic waves of other frequencies confirmed that light was an electromagnetic wave as well.

The frequency of an electromagnetic wave is related to its wavelength:

$$c = \lambda f$$

# 2.2. Light as an Electromagnetic Wave and the Electromagnetic Spectrum

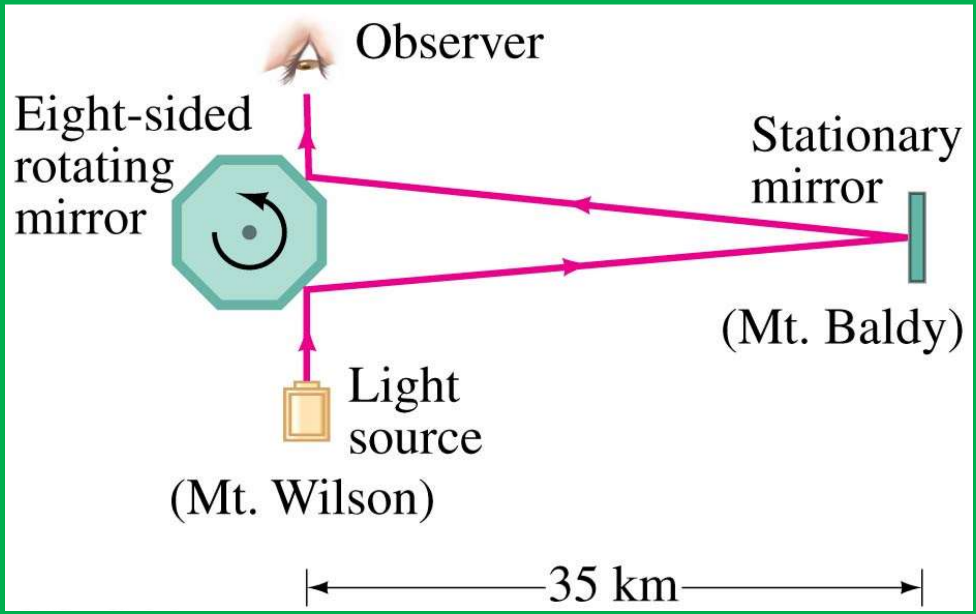
Electromagnetic waves can have any wavelength; we have given different names to different parts of the electromagnetic spectrum.



# 2.3. Measuring the Speed of Light

The speed of light was known to be very large, although careful studies of the orbits of Jupiter's moons showed that it is finite.

One important measurement, by Michelson, used a rotating mirror:



# 2.3. Measuring the Speed of Light

Over the years, measurements have become more and more precise; now the speed of light is defined to be:

$$c = 2.99792458 \times 10^8 \text{ m/s}$$

This is then used to define the meter.

# 2.4. Energy in EM Waves

Energy is stored in both electric and magnetic fields, giving the total energy density of an electromagnetic wave:

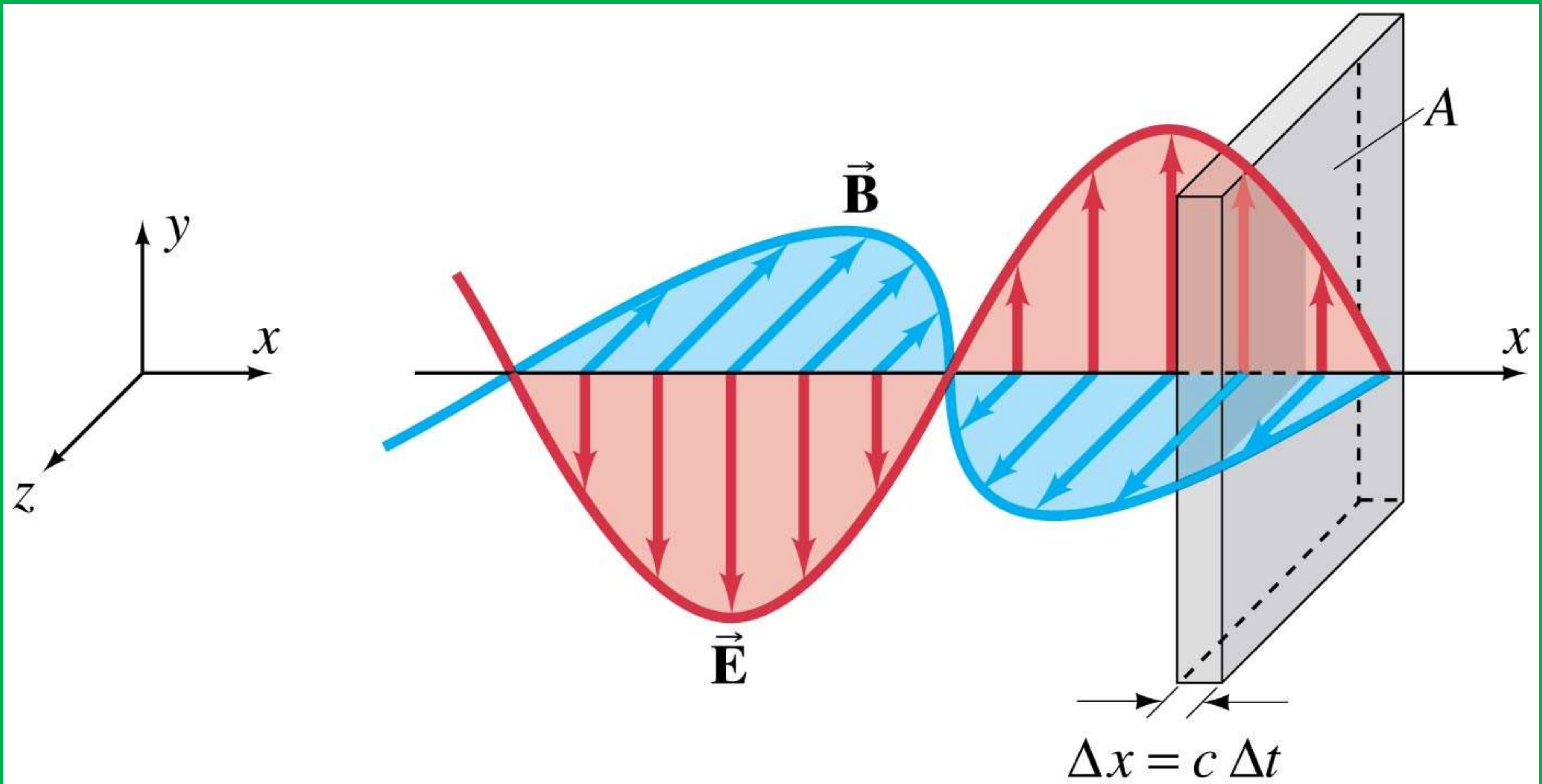
$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}.$$

Each field contributes half the total energy density.

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\epsilon_0 \mu_0 E^2}{\mu_0} = \epsilon_0 E^2.$$

# 2.4. Energy in EM Waves

This energy is transported by the wave.



# 2.4. Energy in EM Waves

The energy transported through a unit area per unit time is called the intensity:

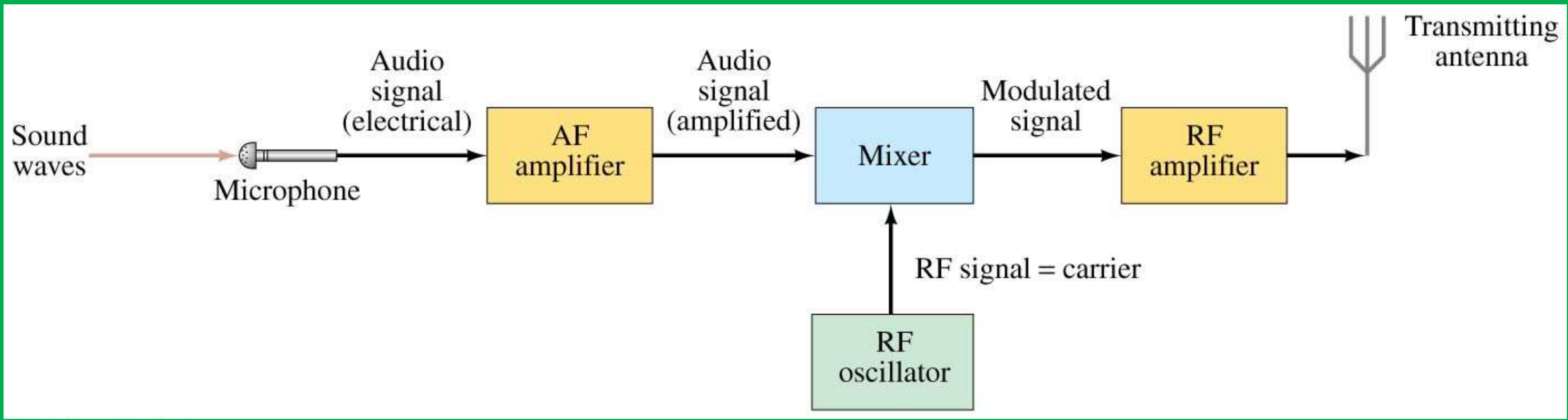
$$I = \epsilon_0 c E^2 = \frac{c}{\mu_0} B^2 = \frac{EB}{\mu_0}.$$

Its average value is given by:

$$\bar{I} = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2\mu_0}.$$

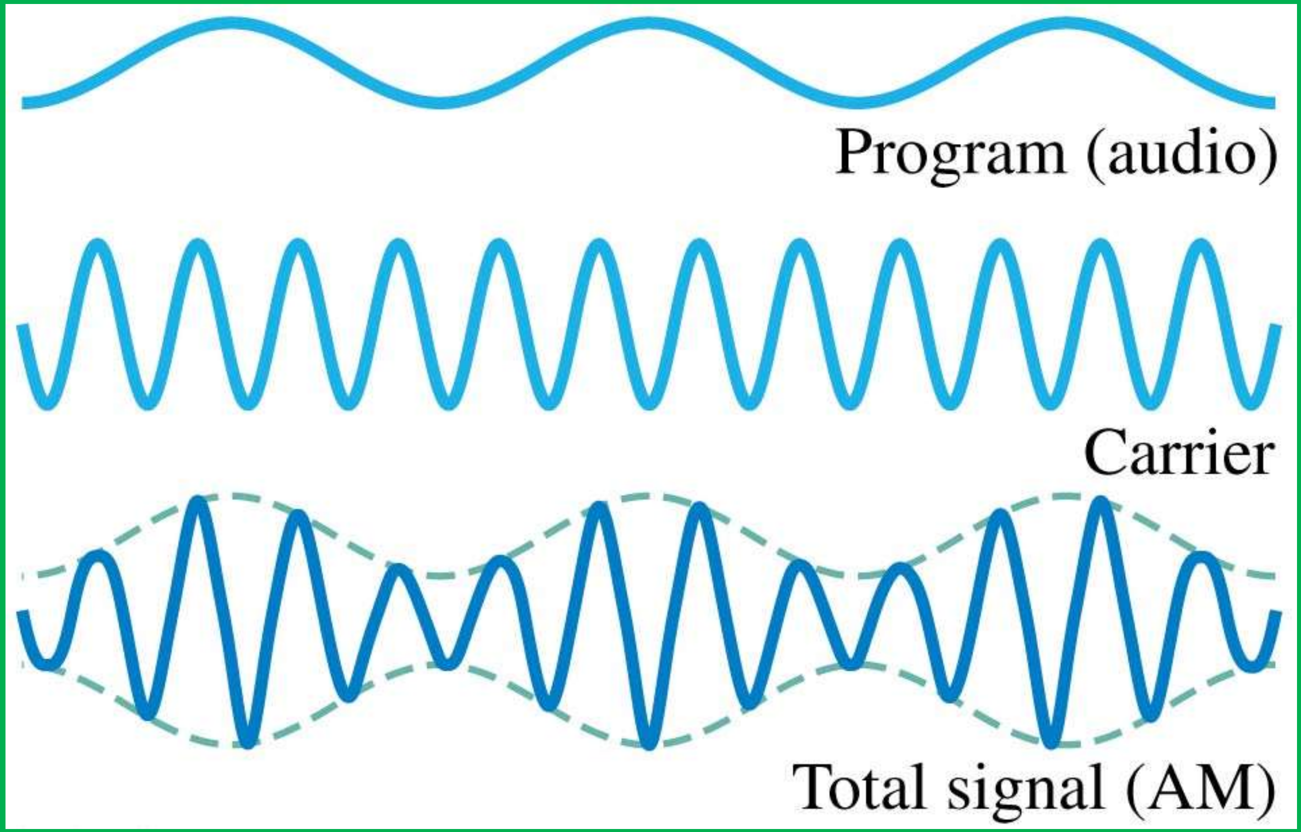
# 2.5. Radio and Television; Wireless Communication

This figure illustrates the process by which a radio station transmits information. The audio signal is combined with a carrier wave:



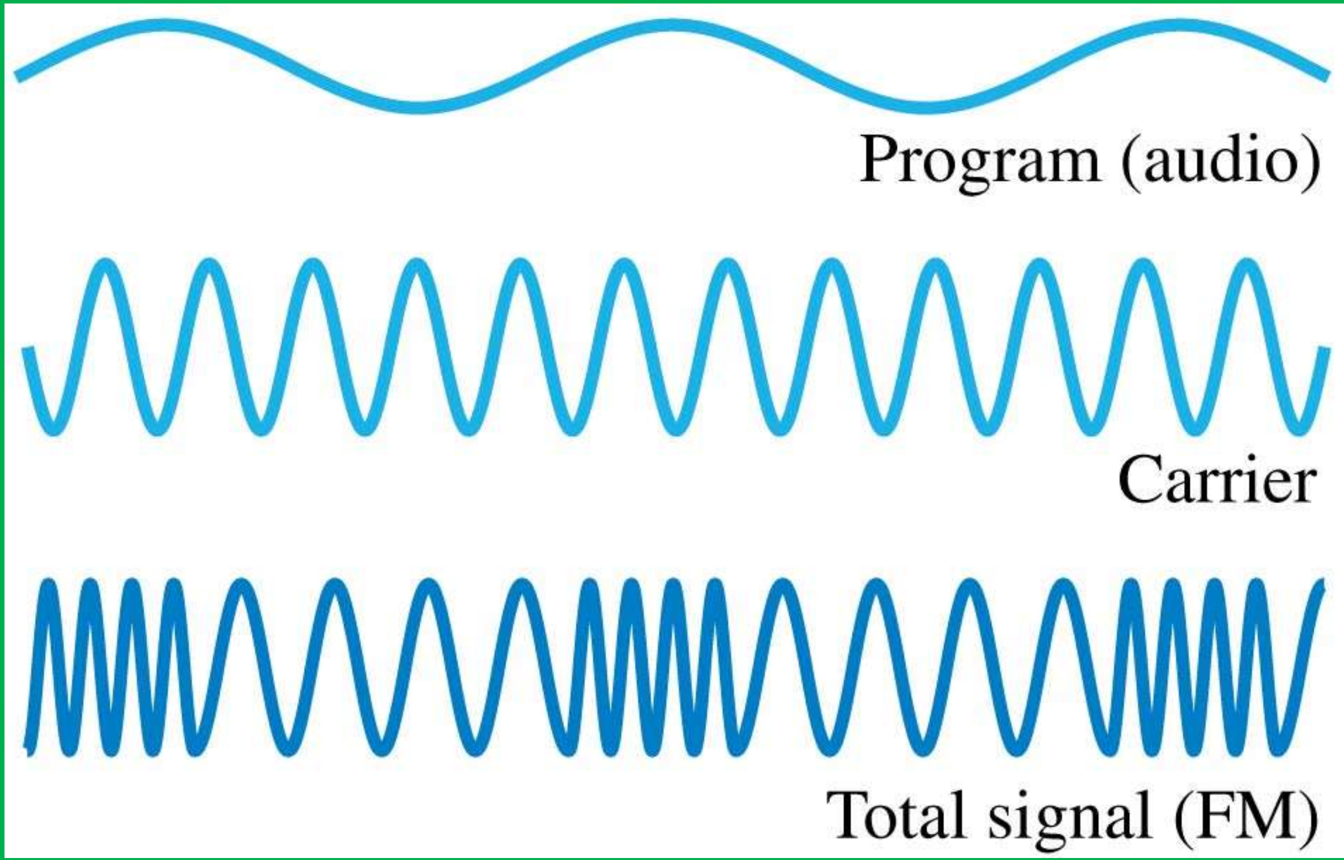
# 2.5. Radio and Television; Wireless Communication

The mixing of signal and carrier can be done two ways. First, by using the signal to modify the amplitude of the carrier (AM):



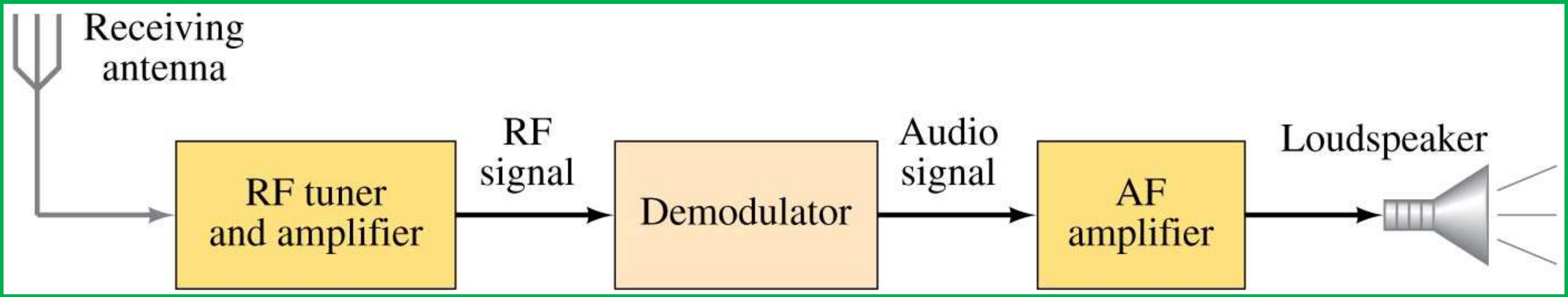
# 2.5. Radio and Television; Wireless Communication

Second, by using the signal to modify the frequency of the carrier (FM):



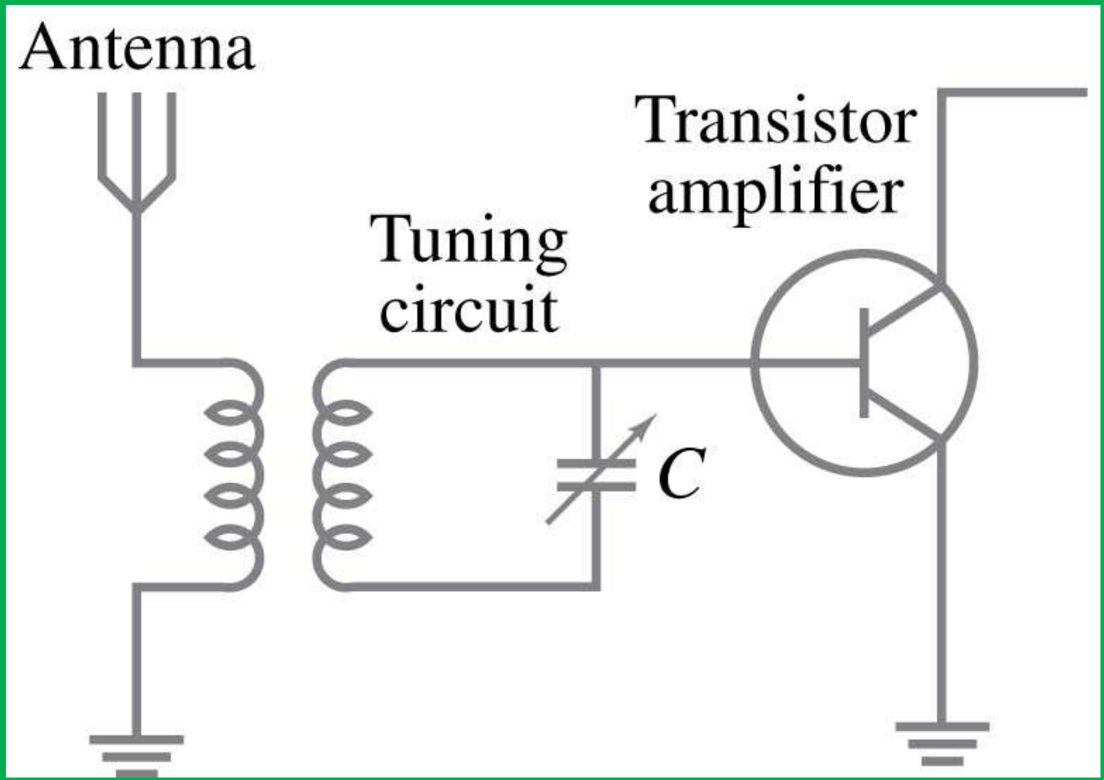
# 2.5. Radio and Television; Wireless Communication

At the receiving end, the wave is received, demodulated, amplified, and sent to a loudspeaker:



# 2.5. Radio and Television; Wireless Communication

The receiving antenna is bathed in waves of many frequencies; a tuner is used to select the desired one:



# 3. Electromagnetic Oscillations and Alternating Current

# 3.1. LC Oscillations

In RC and RL circuits the charge, current, and potential difference grow and decay exponentially.

On the contrary, in an LC circuit, the charge, current, and potential difference vary sinusoidally with period $T$ and angular frequency $\omega$.

The resulting oscillations of the capacitor's electric field and the inductor's magnetic field are said to be **electromagnetic oscillations.**

# 3.1. LC Oscillations

The energy stored in the electric field of the capacitor at any time is $U_E = \frac{q^2}{2C}$, where $q$ is the charge on the capacitor at that time.

The energy stored in the magnetic field of the inductor at any time is $U_B = \frac{Li^2}{2}$, where $i$ is the current through the inductor at that time.

As the circuit oscillates, energy shifts back and forth from one type of stored energy to the other, but the total amount is conserved.

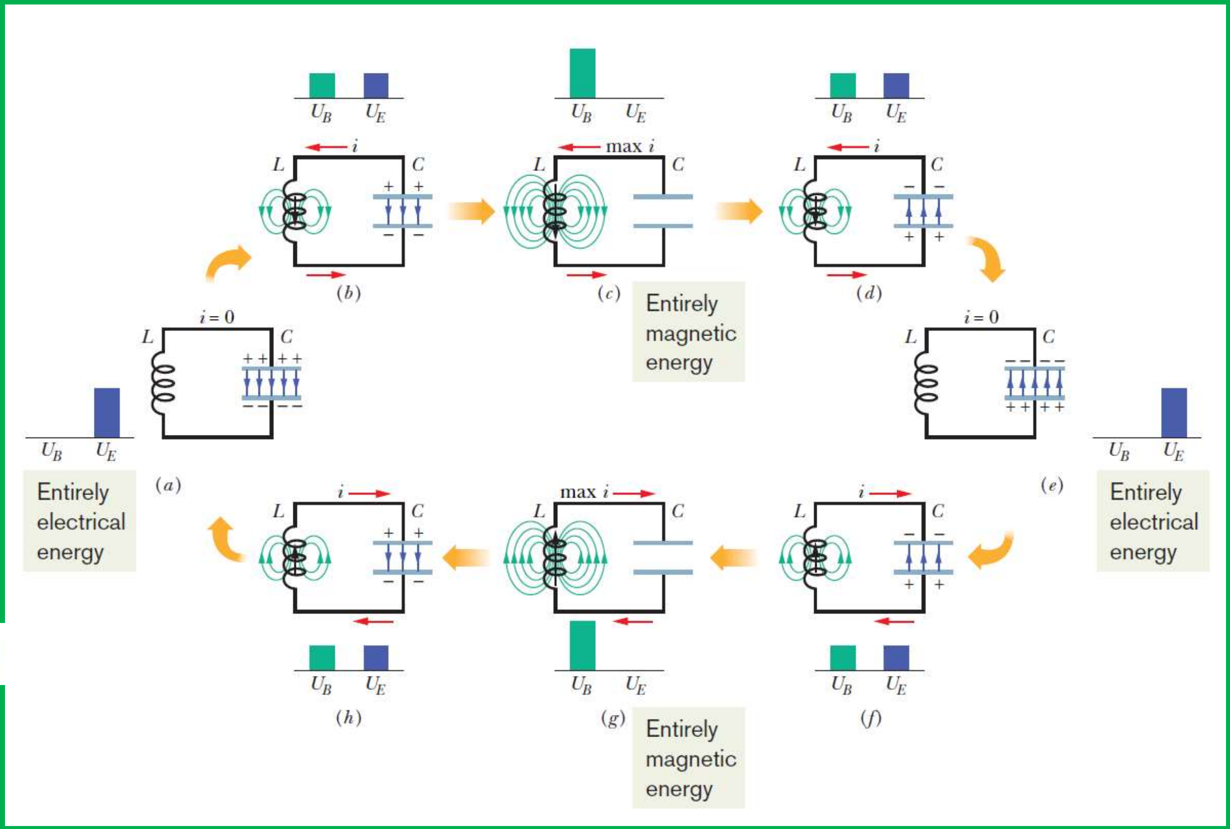



Fig. 31-1

# 3.1. LC Oscillations

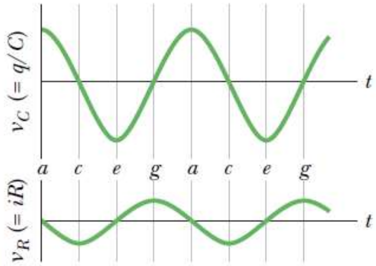
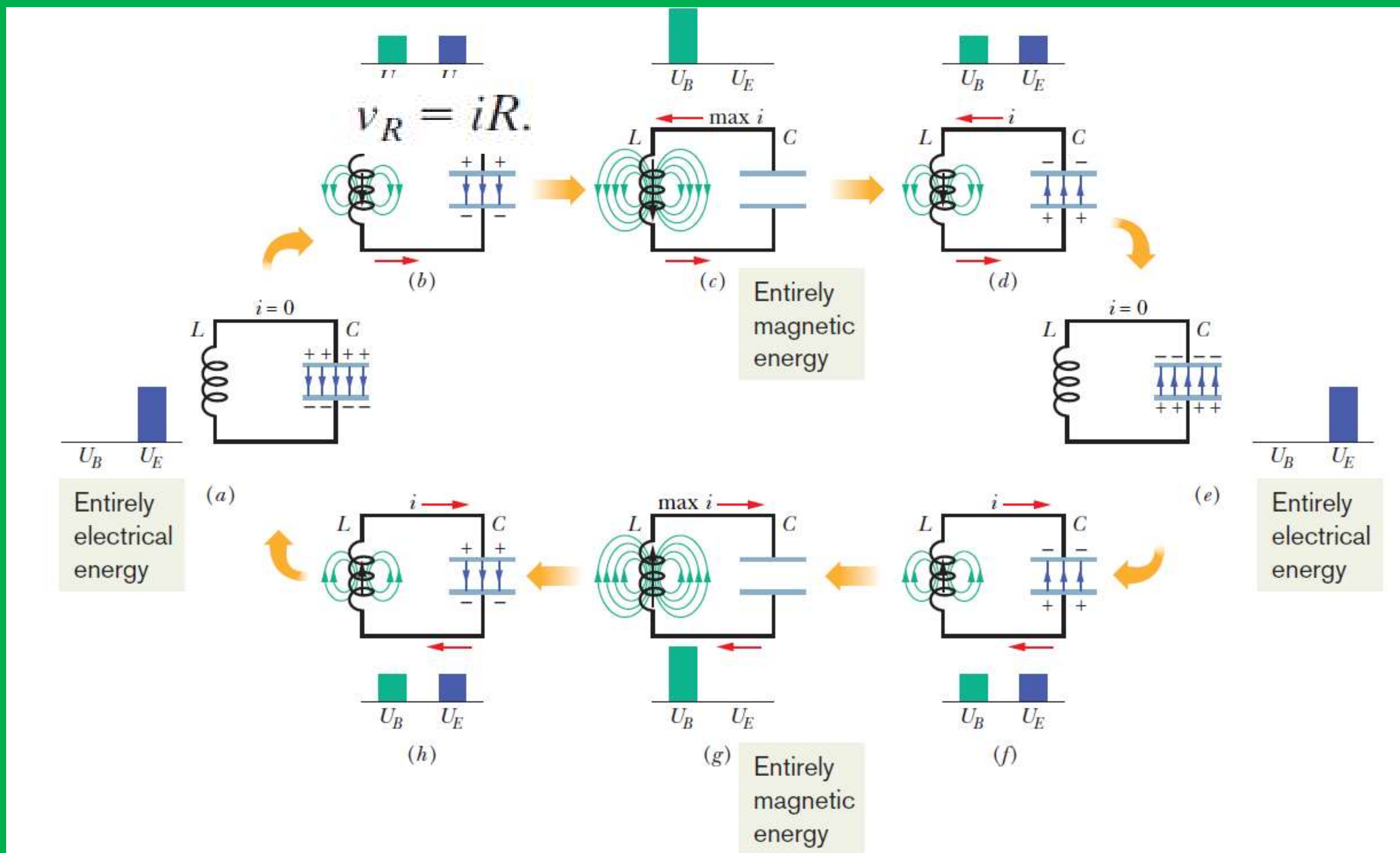



**Fig. 31-2** (*a*) The potential difference across the capacitor of the circuit of Fig. 31-1 as a function of time. This quantity is proportional to the charge on the capacitor. (*b*) A potential proportional to the current in the circuit of Fig. 31-1. The letters refer to the correspondingly labeled oscillation stages in Fig. 31-1.

The time-varying potential difference (or *voltage)* $v_C$ that exists across the capacitor $C$ is $v_C = \left(\frac{1}{C}\right) q,$

To measure the current, we can connect a small resistance $R$ in series with the capacitor and inductor and measure the time-varying potential difference $v_R$ across it: $v_R = iR.$



# 3.1. LC Oscillations

One can make an analogy between the oscillating *LC* system and an oscillating block–spring system.

Two kinds of energy are involved in the block–spring system. One is potential energy of the compressed or extended spring; the other is kinetic energy of the moving block.

Here we have the following analogies:

$q$ corresponds to $x$, $\quad$ $1/C$ corresponds to $k$,

$i$ corresponds to $v$, $\quad$ and $\quad$ $L$ corresponds to $m$.

**Table 31-1**

**Comparison of the Energy in Two Oscillating Systems**

| Block–Spring System | | *LC* Oscillator | |
|---|---|---|---|
| Element | Energy | Element | Energy |
| Spring | Potential, $\frac{1}{2}kx^2$ | Capacitor | Electrical, $\frac{1}{2}(1/C)q^2$ |
| Block | Kinetic, $\frac{1}{2}mv^2$ | Inductor | Magnetic, $\frac{1}{2}Li^2$ |
| $v = dx/dt$ | | $i = dq/dt$ | |

The angular frequency of oscillation for an ideal (resistanceless) *LC* is:

$$\omega = \frac{1}{\sqrt{LC}} \qquad (LC \text{ circuit}).$$

# 3.1. LC Oscillations

**The Block-Spring Oscillator:**

$$U = U_b + U_s = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2,$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0.$$

$$m\frac{d^2x}{dt^2} + kx = 0 \quad \rightarrow \quad x = X\cos(\omega t + \phi) \qquad \text{(displacement)}$$

**The LC Oscillator:**

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C},$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0.$$

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \qquad (LC\ \text{oscillations}).$$

$$q = Q\cos(\omega t + \phi) \qquad \text{(charge)}, \quad \rightarrow \quad i = \frac{dq}{dt} = -\omega Q\sin(\omega t + \phi) \qquad \text{(current)}.$$

$$I = \omega Q, \quad \rightarrow \quad i = -I\sin(\omega t + \phi).$$

# 3.1. LC Oscillations

**Angular Frequencies:**

$$\omega = \frac{1}{\sqrt{LC}}.$$

# 3.1. LC Oscillations

The electrical energy stored in the *LC* circuit at time *t* is,

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi).$$

The magnetic energy is:

$$U_B = \tfrac{1}{2}Li^2 = \tfrac{1}{2}L\omega^2Q^2\sin^2(\omega t + \phi).$$

But

$$\omega = \frac{1}{\sqrt{LC}} \qquad (LC \text{ circuit}).$$

Therefore

$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

Note that

- The maximum values of $U_E$ and $U_B$ are both $Q^2/2C$.
- At any instant the sum of $U_E$ and $U_B$ is equal to $Q^2/2C$, a constant.
- When $U_E$ is maximum, $U_B$ is zero, and conversely.

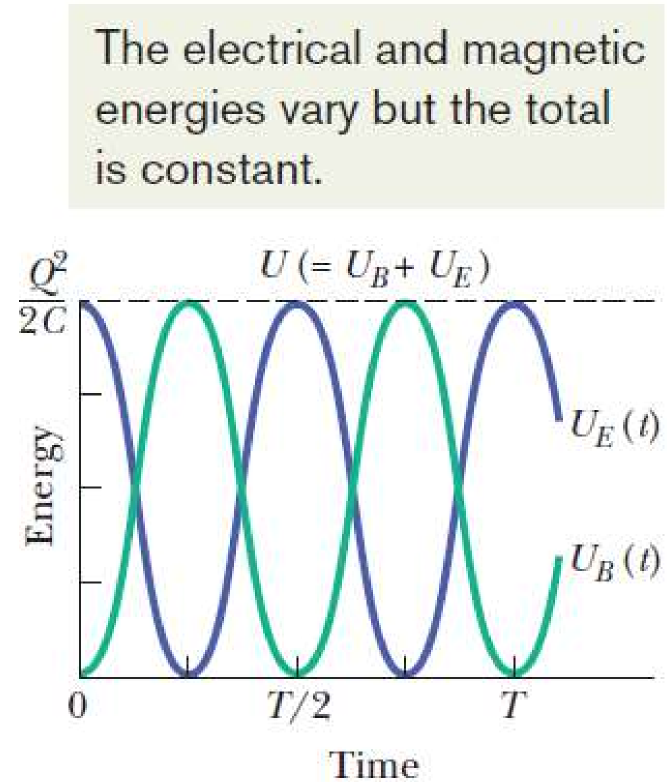
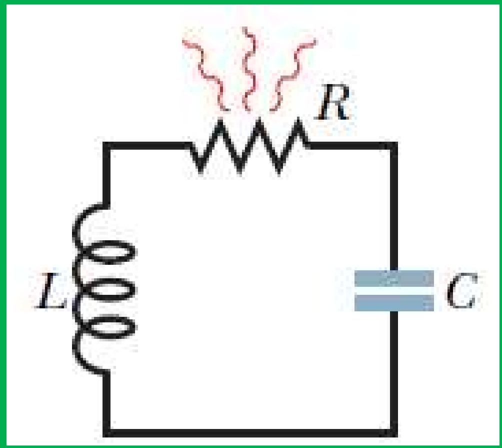



**Fig. 31-4** The stored magnetic energy and electrical energy in the circuit of Fig. 31-1 as a function of time. Note that their sum remains constant. *T* is the period of oscillation.

# 3.2. Damped Oscillations in an RLC Circuit



# 3.2. Damped Oscillations in an RLC Circuit

**Analysis:**

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

$$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2R.$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \qquad (RLC\text{ circuit}),$$

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi),$$

Where $\omega' = \sqrt{\omega^2 - (R/2L)^2},$

And $\omega = 1/\sqrt{LC}$

$$U_E = \frac{q^2}{2C} = \frac{[Qe^{-Rt/2L}\cos(\omega' t + \phi)]^2}{2C} = \frac{Q^2}{2C}e^{-Rt/L}\cos^2(\omega' t + \phi)$$

# 3.3. Alternating Current

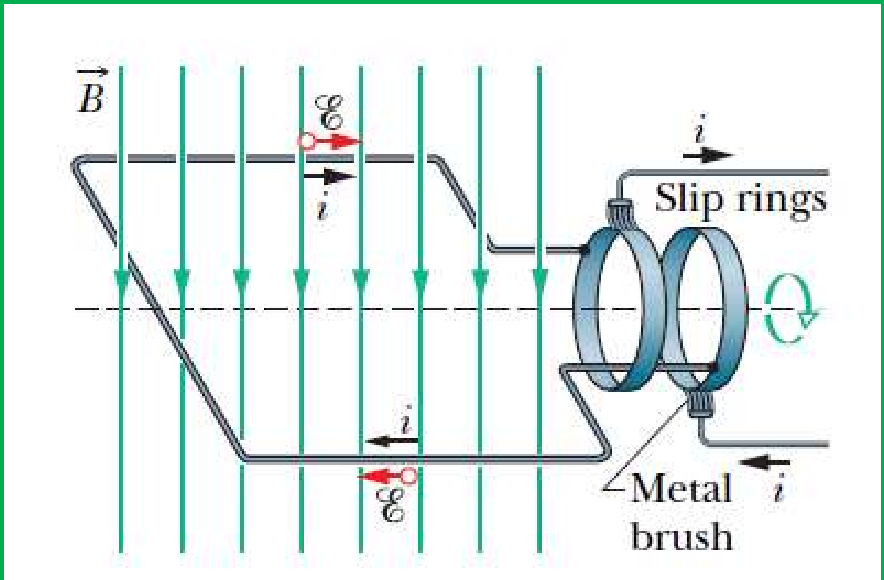



**Fig. 31-6** The basic mechanism of an alternating-current generator is a conducting loop rotated in an external magnetic field. In practice, the alternating emf induced in a coil of many turns of wire is made accessible by means of slip rings attached to the rotating loop. Each ring is connected to one end of the loop wire and is electrically connected to the rest of the generator circuit by a conducting brush against which the ring slips as the loop (and it) rotates.

$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t.$$

$$i = I \sin(\omega_d t - \phi),$$

$\omega_d$ is called the driving angular frequency, and $I$ is the amplitude of the driven current.

# 3.4. Forced Oscillations:

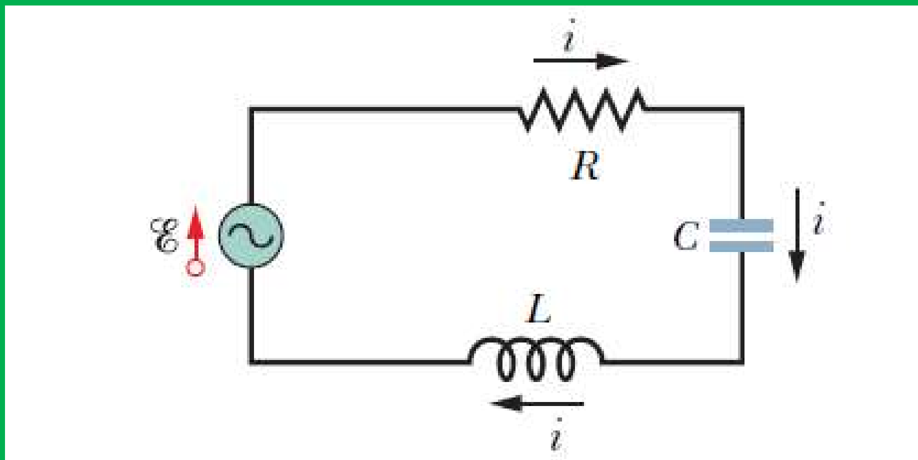



**Fig. 31-7** A single-loop circuit containing a resistor, a capacitor, and an inductor. A generator, represented by a sine wave in a circle, produces an alternating emf that establishes an alternating current; the directions of the emf and current are indicated here at only one instant.

Whatever the natural angular frequency $\omega$ of a circuit may be, forced oscillations of charge, current, and potential difference in the circuit always occur at the driving angular frequency $\omega_d$.

# 3.5. Three Simple Circuits

## *i. A Resistive Load:*

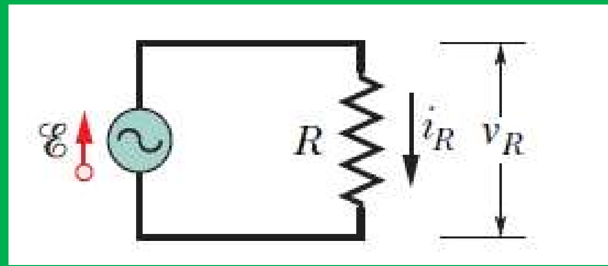



**Fig. 31-8** A resistor is connected across an alternating-current generator.

$$\mathscr{E} - v_R = 0.$$

$$v_R = \mathscr{E}_m \sin \omega_d t. = V_R \sin \omega_d t.$$

$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin \omega_d t.$$

$$= I_R \sin(\omega_d t - \phi),$$

For a purely resistive load the phase constant $\phi = 0°$.

# 3.5. Three Simple Circuits

## i. A Resistive Load:

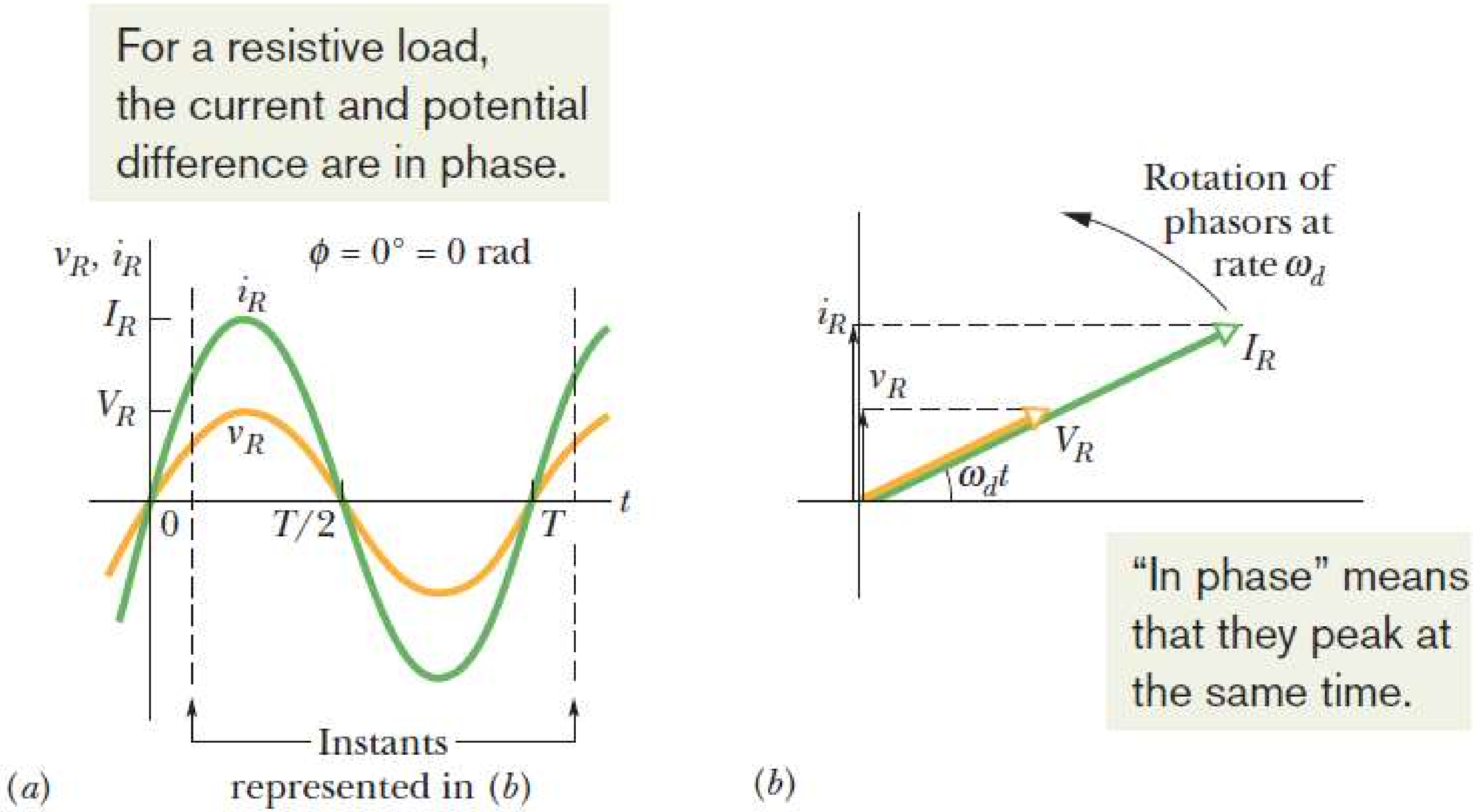



**Fig. 31-9** (*a*) The current $i_R$ and the potential difference $v_R$ across the resistor are plotted on the same graph, both versus time $t$. They are in phase and complete one cycle in one period $T$. (*b*) A phasor diagram shows the same thing as (*a*).

# 3.5. Three Simple Circuits

## *ii. A Capacitive Load:*

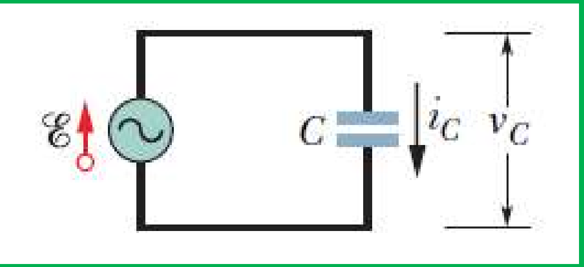



**Fig. 31-10** A capacitor is connected across an alternating-current generator.

$$v_C = V_C \sin \omega_d t,$$

$$q_C = C v_C = C V_C \sin \omega_d t.$$

$$i_C = \frac{dq_C}{dt} = \omega_d C V_C \cos \omega_d t.$$

$$X_C = \frac{1}{\omega_d C} \qquad \text{(capacitive reactance).}$$

$X_C$ is called the **capacitive reactance of a capacitor.** The SI unit of $X_C$ is the *ohm,* just as for resistance *R*.

# 3.5. Three Simple Circuits

## *ii. A Capacitive Load:*

$$\cos \omega_d t = \sin(\omega_d t + 90^\circ).$$

$$i_C = \left(\frac{V_C}{X_C}\right) \sin(\omega_d t + 90^\circ).$$

$$i_C = I_C \sin(\omega_d t - \phi),$$

$$V_C = I_C X_C \qquad \text{(capacitor).}$$

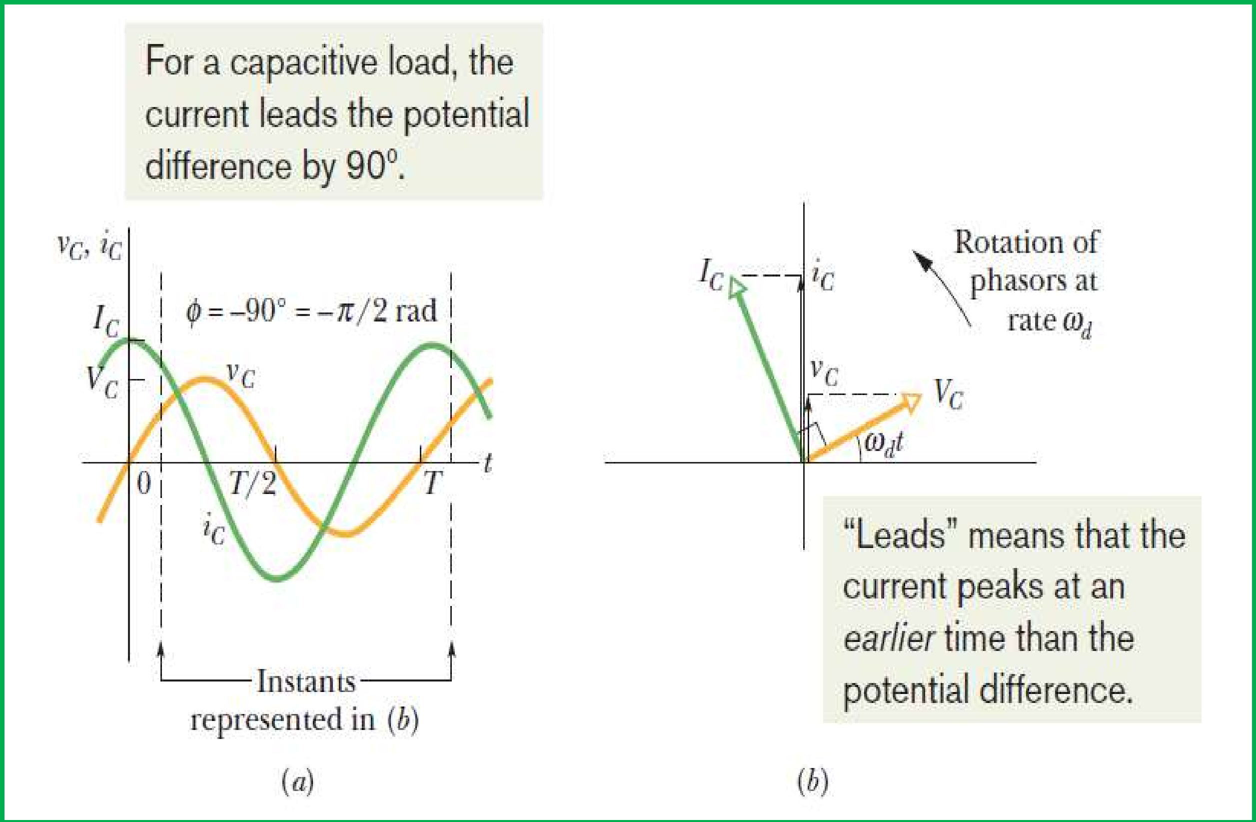



**Fig. 31-11** (*a*) The current in the capacitor leads the voltage by 90° (= $\pi/2$ rad). (*b*) A phasor diagram shows the same thing.

# 3.5. Three Simple Circuits

## iii. An Inductive Load:

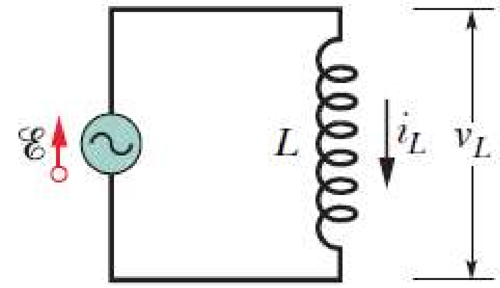



**Fig. 31-12** An inductor is connected across an alternating-current generator.

$$v_L = V_L \sin \omega_d t,$$

$$v_L = L \frac{di_L}{dt}.$$

$$\frac{di_L}{dt} = \frac{V_L}{L} \sin \omega_d t.$$

$$i_L = \int di_L = \frac{V_L}{L} \int \sin \omega_d t \, dt = -\left(\frac{V_L}{\omega_d L}\right) \cos \omega_d t.$$

$$X_L = \omega_d L \qquad \text{(inductive reactance).}$$

$$i_L = \left(\frac{V_L}{X_L}\right) \sin(\omega_d t - 90°).$$

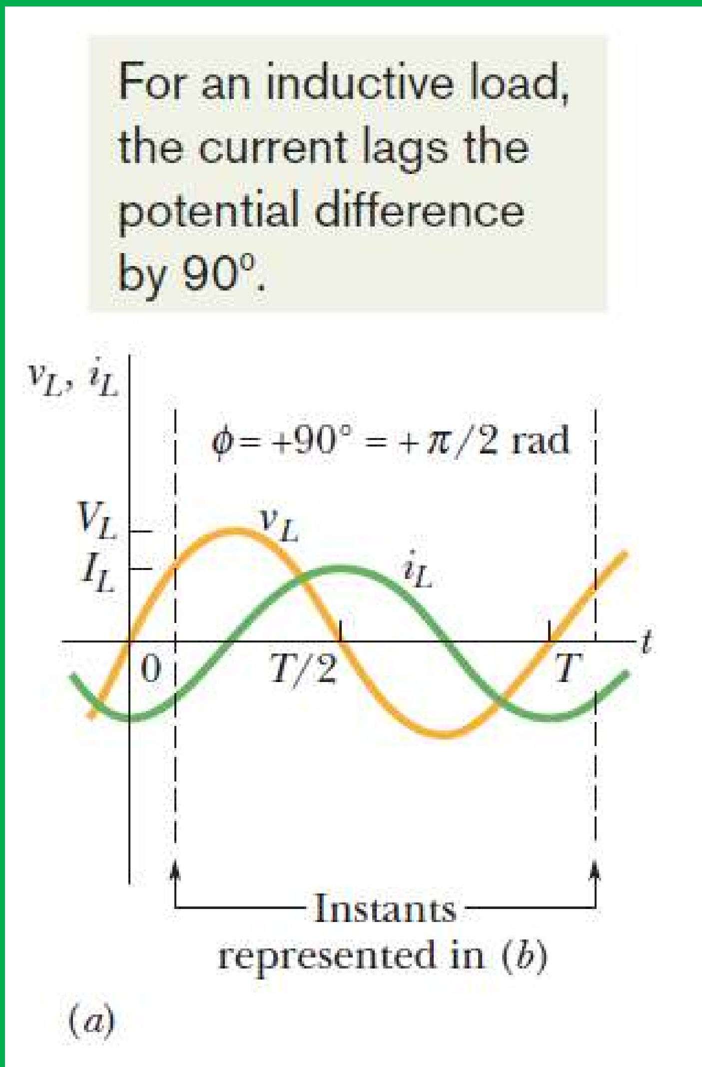
$$i_L = I_L \sin(\omega_d t - \phi),$$

$$V_L = I_L X_L \qquad \text{(inductor).}$$

The value of $X_L$ , **the inductive resistance**, depends on the driving angular frequency $\omega_d$. The unit of the inductive time constant $\tau_L$ indicates that the SI unit of $X_L$ is the *ohm*.

# 3.5. Three Simple Circuits

## iii. An Inductive Load:



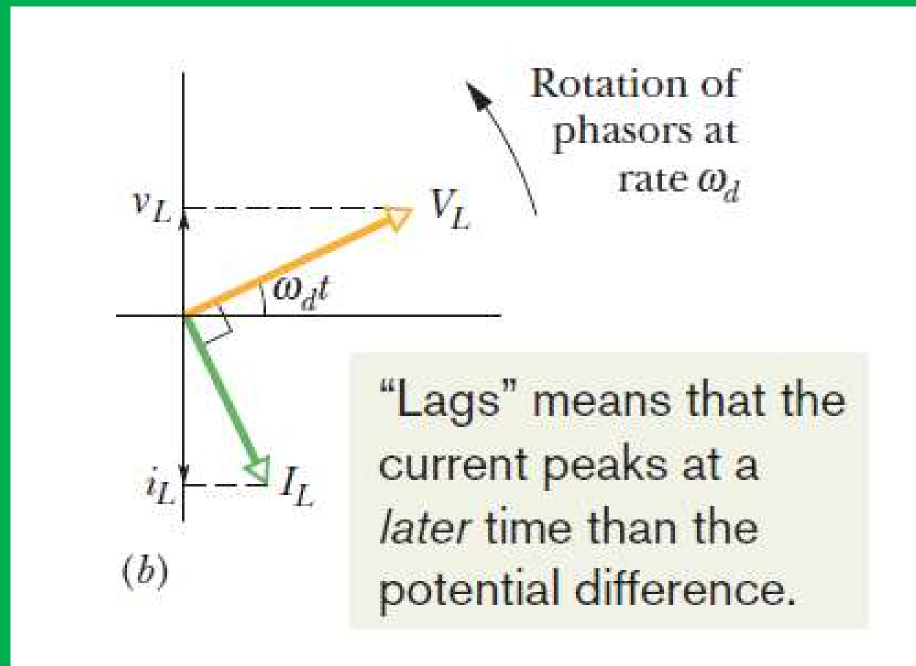




**Fig. 31-13** (*a*) The current in the inductor lags the voltage by 90° (= $\pi/2$ rad). (*b*) A phasor diagram shows the same thing.

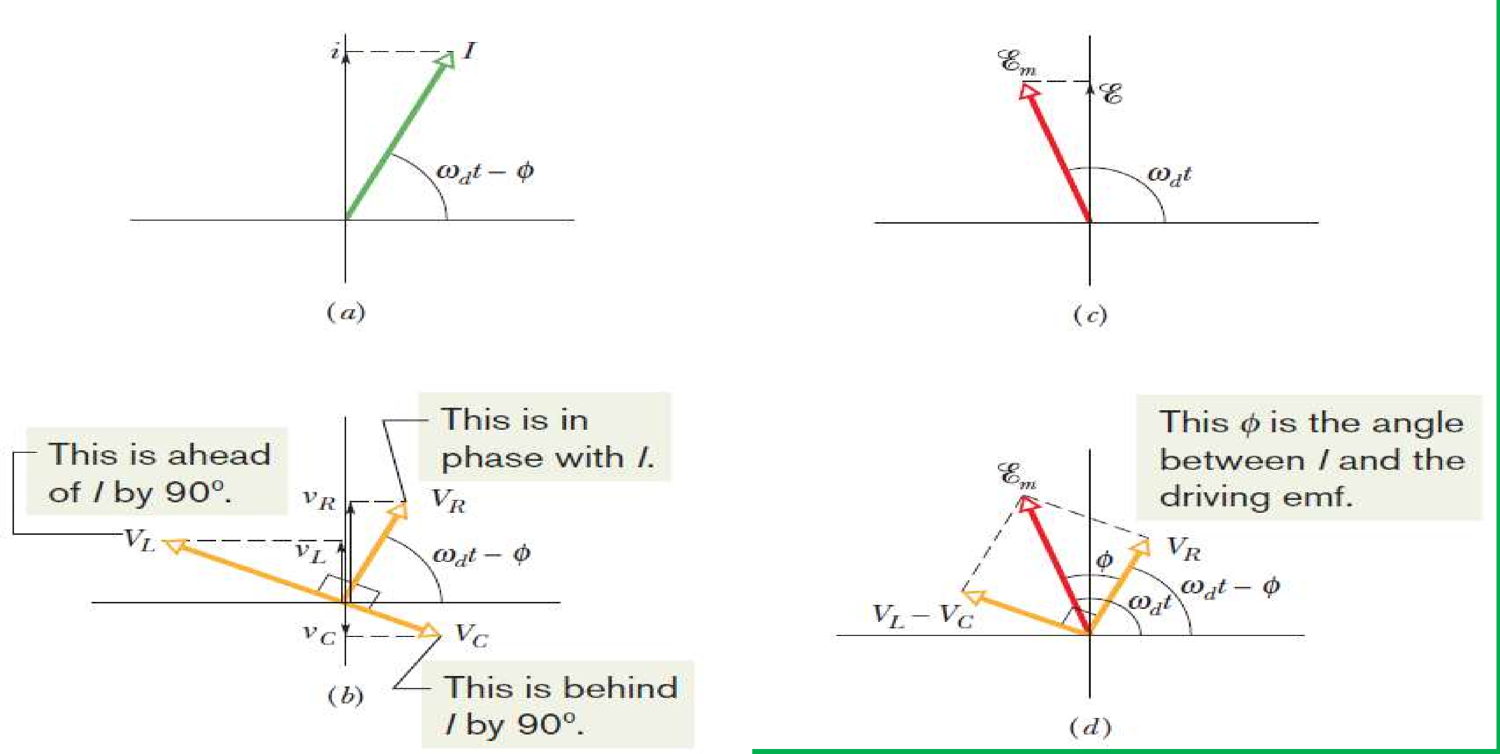
# 3.6. Three Simple Circuits

**Table 31-2**

**Phase and Amplitude Relations for Alternating Currents and Voltages**

| Circuit Element | Symbol | Resistance or Reactance | Phase of the Current | Phase Constant (or Angle) $\phi$ | Amplitude Relation |
|---|---|---|---|---|---|
| Resistor | $R$ | $R$ | In phase with $v_R$ | $0°$ (= 0 rad) | $V_R = I_R R$ |
| Capacitor | $C$ | $X_C = 1/\omega_d C$ | Leads $v_C$ by $90°$ (= $\pi/2$ rad) | $-90°$ (= $-\pi/2$ rad) | $V_C = I_C X_C$ |
| Inductor | $L$ | $X_L = \omega_d L$ | Lags $v_L$ by $90°$ (= $\pi/2$ rad) | $+90°$ (= $+\pi/2$ rad) | $V_L = I_L X_L$ |

# 3.6. Three Simple Circuits:



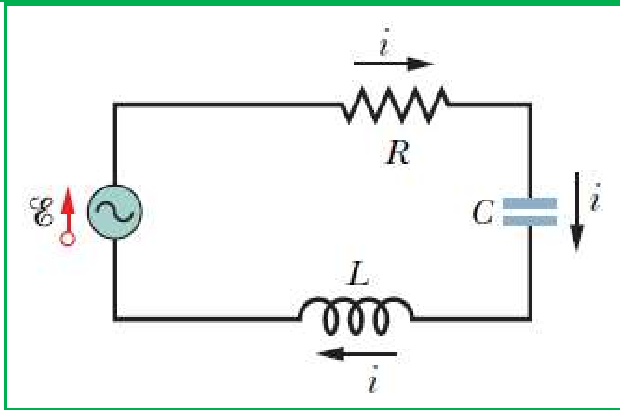
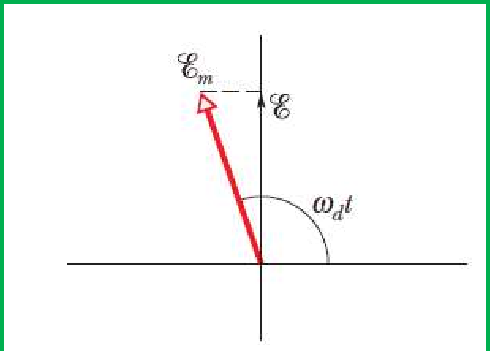



**Fig. 31-14** (*a)* A phasor representing the alternating current in the driven *RLC* circuit at time *t*. The amplitude *I,* the instantaneous value *i,* and the phase$(\omega_d t\text{-}\phi)$ are shown.
(*b)* Phasors representing the voltages across the inductor, resistor, and capacitor, oriented with respect to the current phasor in (*a)*.
*(c)* A phasor representing the alternating emf that drives the current of (*a)*.
(*d)* The emf phasor is equal to the vector sum of the three voltage phasors of (*b)*.Here, voltage phasors $V_L$ and $V_C$ have been added vectorially to yield their net phasor $(V_L\text{-}V_C)$.

# 3.7. The Series RLC Circuit:




$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t$$

$$i = I \sin(\omega_d t - \phi)$$

$$\mathscr{E}_m^2 = V_R^2 + (V_L - V_C)^2. = (IR)^2 + (IX_L - IX_C)^2,$$

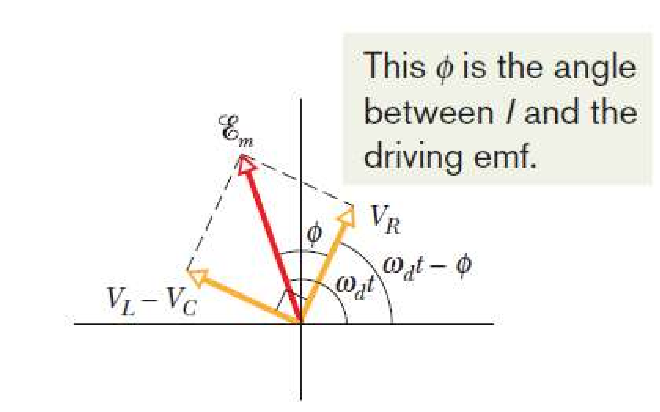
$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}.$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{(impedance defined).}$$

$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} \qquad \text{(current amplitude).}$$

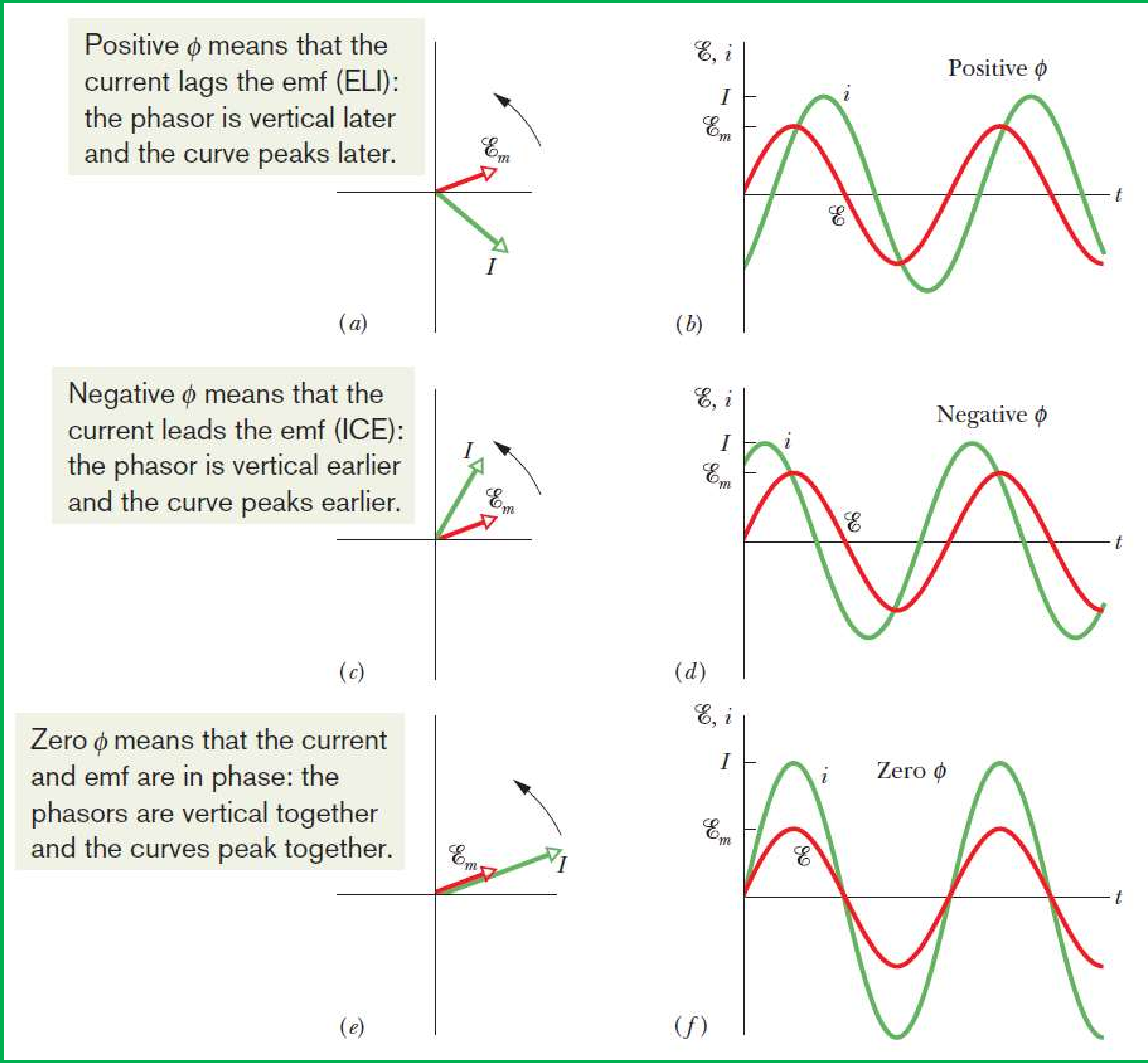
$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR},$$

$$\tan \phi = \frac{X_L - X_C}{R} \qquad \text{(phase constant).}$$



# 3.7. The Series RLC Circuit:

**Fig. 31-15** Phasor diagrams and graphs of the alternating emf and current *i* for a driven *RLC* circuit. In the phasor diagram of *(a)* and the graph of *(b),* the current *I* lags the driving emf and the current's phase constant $\phi$ is positive. In *(c)* and *(d),* the current *i* leads the driving emf and its phase constant $\phi$ is negative. In *(e)* and *( f ),* the current *i* is in phase with the driving emf and its phase constant $\phi$ is zero.



# 3.7. The Series RLC Circuit, Resonance

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} \quad \text{(current amplitude).}$$

For a given resistance *R,* that amplitude is a maximum when the quantity $(\omega_d L - 1/\omega_d C)$ in the denominator is zero.

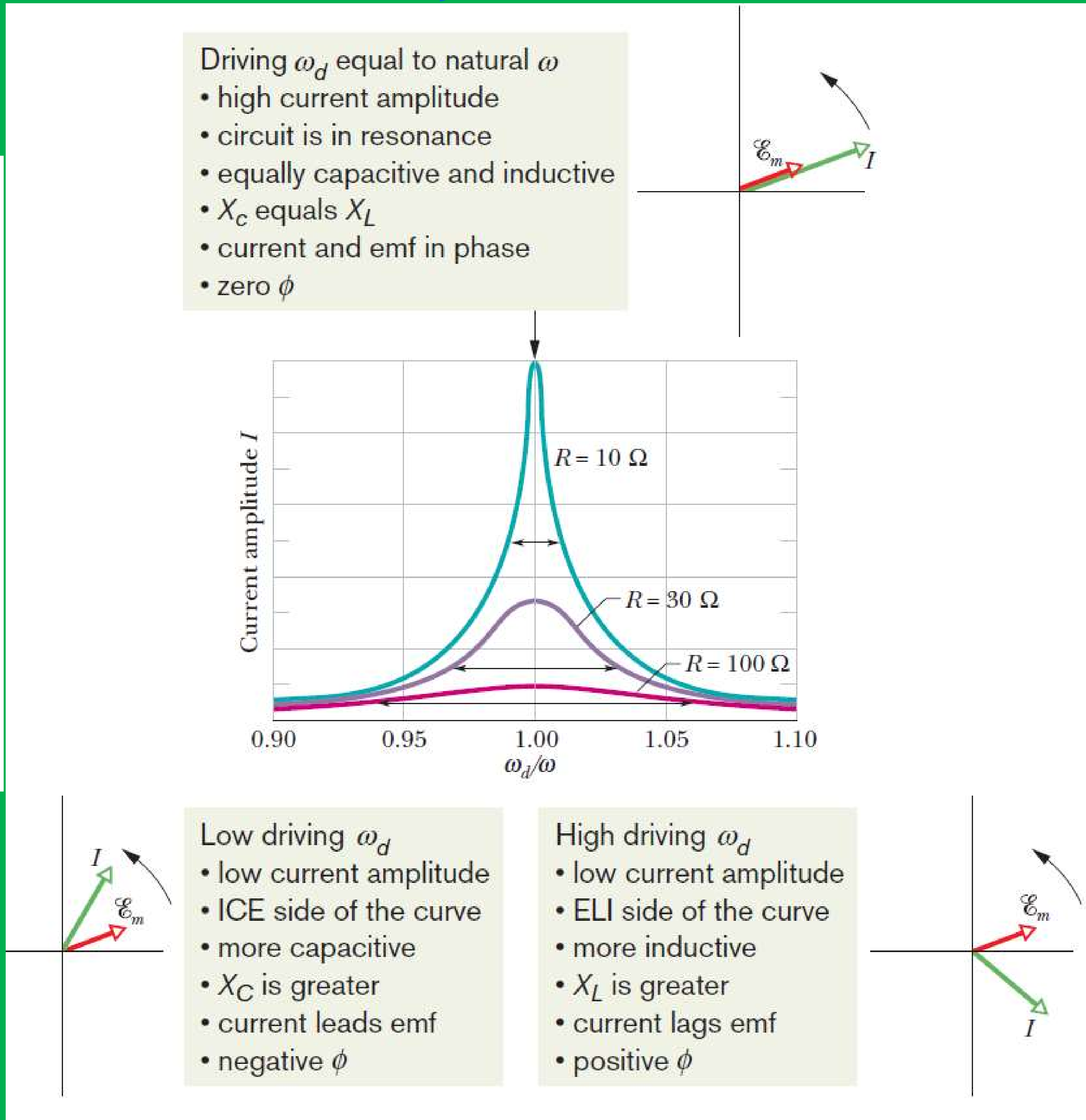
$$\Rightarrow \quad \omega_d L = \frac{1}{\omega_d C} \quad \Rightarrow \quad \omega_d = \frac{1}{\sqrt{LC}} \quad \text{(maximum } I\text{).}$$

The maximum value of *I* occurs when the driving angular frequency matches the natural angular frequency—that is, at resonance.

$$\omega_d = \omega = \frac{1}{\sqrt{LC}} \quad \text{(resonance).}$$

# 3.7. The Series RLC Circuit, Resonance :

**Fig. 31-16** *Resonance curves* for the driven $RLC$ circuit of Fig. 31-7 with $L = 100\ \mu H$, $C = 100$ pF, and three values of $R$. The current amplitude $I$ of the alternating current depends on how close the driving angular frequency $\omega_d$ is to the natural angular frequency $\omega$. The horizontal arrow on each curve measures the curve's *half-width,* which is the width at the half-maximum level and is a measure of the sharpness of the resonance. To the left of $\omega_d/\omega = 1.00$, the circuit is mainly capacitive, with $X_C > X_L$; to the right, it is mainly inductive, with $X_L > X_C$.



# 3.8. Power in Alternating Current Circuits

The instantaneous rate at which energy is dissipated in the resistor:

$$P = i^2R = [I\sin(\omega_d t - \phi)]^2 R = I^2 R \sin^2(\omega_d t - \phi).$$

The average rate at which energy is dissipated in the resistor, is the average of this over time:

$$P_{\text{avg}} = \frac{I^2R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R.$$

Since the root mean square of the current is given by:

Similarly, $$I_{\text{rms}} = \frac{I}{\sqrt{2}}$$ $$P_{\text{avg}} = I_{\text{rms}}^2 R \qquad \text{(average power).}$$

With $$V_{\text{rms}} = \frac{V}{\sqrt{2}} \quad \text{and} \quad \mathscr{E}_{\text{rms}} = \frac{\mathscr{E}_m}{\sqrt{2}} \qquad \text{(rms voltage; rms emf).}$$

Therefore, $$I_{\text{rms}} = \frac{\mathscr{E}_{\text{rms}}}{Z} = \frac{\mathscr{E}_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}},$$

$$P_{\text{avg}} = \frac{\mathscr{E}_{\text{rms}}}{Z} I_{\text{rms}} R = \mathscr{E}_{\text{rms}} I_{\text{rms}} \frac{R}{Z}.$$

$$P_{\text{avg}} = \mathscr{E}_{\text{rms}} I_{\text{rms}} \cos\phi \qquad \text{(average power),}$$

where $$\cos\phi = \frac{V_R}{\mathscr{E}_m} = \frac{IR}{IZ} = \frac{R}{Z}.$$

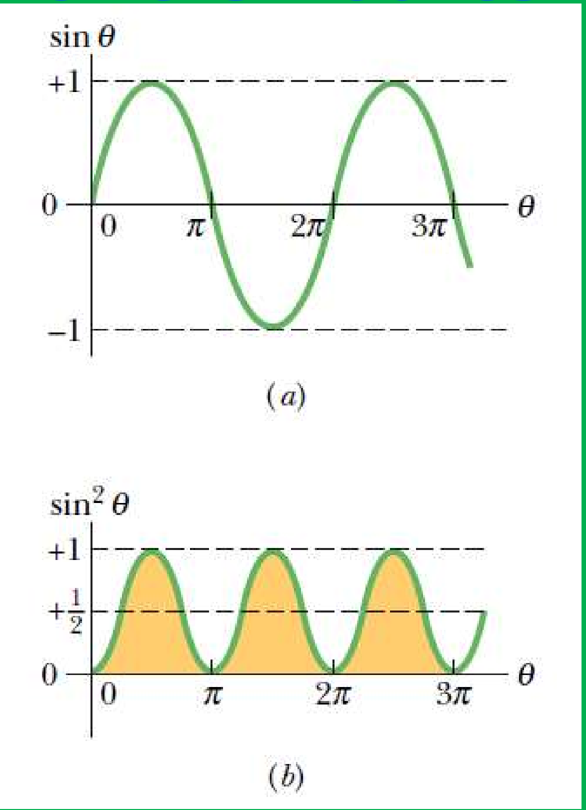



**Fig. 31-17** (*a*) A plot of $\sin\theta$ versus $\theta$. The average value over one cycle is zero. (*b*) A plot of $\sin^2\theta$ versus $\theta$. The average value over one cycle is $\frac{1}{2}$.

# 3.9. Transformers

In electrical power distribution systems it is desirable for reasons of safety and for efficient equipment design to deal with relatively low voltages at both the generating end (the electrical power plant) and the receiving end (the home or factory).

Nobody wants an electric toaster or a child's electric train to operate at, say, 10 kV.

On the other hand, in the transmission of electrical energy from the generating plant to the consumer, we want the lowest practical current (hence the largest practical voltage) to minimize $I^2R$ losses (often called ohmic losses) in the transmission line.

# 3.9. Transformers

A device with which we can raise and lower the ac voltage in a circuit, keeping the product current voltage essentially constant, is called the **transformer.**

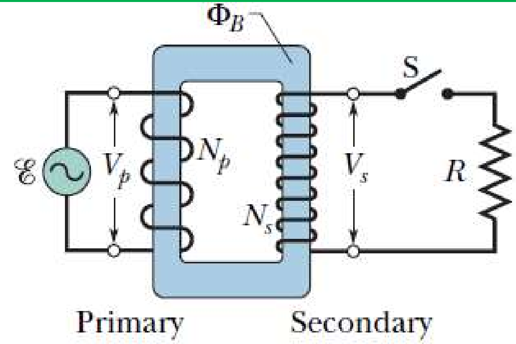



**Fig. 31-18** An ideal transformer (two coils wound on an iron core) in a basic transformer circuit. An ac generator produces current in the coil at the left (the *primary*). The coil at the right (the *secondary*) is connected to the resistive load $R$ when switch S is closed.

The ideal transformer consists of two coils, with different numbers of turns, wound around an iron core.

In use, the primary winding, of $N_p$ turns, is connected to an alternating-current generator whose emf at any time $t$ is given by $\mathscr{E} = \mathscr{E}_m \sin \omega t.$

The secondary winding, of $N_s$ turns, is connected to load resistance $R$, but its circuit is an open circuit as long as switch S is open.

The small sinusoidally changing primary current $I_{mag}$ produces a sinusoidally changing magnetic flux $B$ in the iron core.

Because $B$ varies, it induces an emf ( $dB/dt$) in each turn of the secondary. This emf per turn is the same in the primary and the secondary. Across the primary, the voltage $V_p = \mathcal{E}_{\text{turn}} N_p$. Similarly, across the secondary the voltage is $V_s = \mathcal{E}_{turn} N_s$.

$$V_s = V_p \frac{N_s}{N_p} \quad \text{(transformation of voltage)}.$$

# 3.9. Transformers

$$V_s = V_p \frac{N_s}{N_p} \qquad \text{(transformation of voltage).}$$

If $N_s > N_p$, the device is a step-up transformer because it steps the primary's voltage $V_p$ up to a higher voltage $V_s$. Similarly, if $N_s < N_p$, it is a step-down transformer.

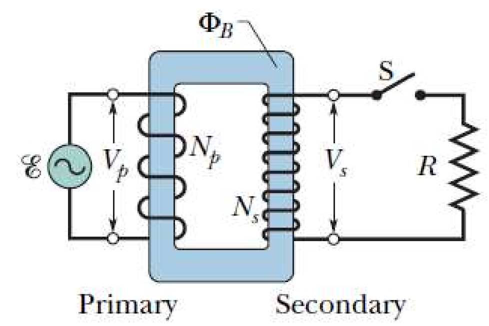



**Fig. 31-18** An ideal transformer (two coils wound on an iron core) in a basic transformer circuit. An ac generator produces current in the coil at the left (the *primary*). The coil at the right (the *secondary*) is connected to the resistive load $R$ when switch S is closed.

If no energy is lost along the way, conservation of energy requires that

$$I_p V_p = I_s V_s. \Rightarrow I_s = I_p \frac{N_p}{N_s} \qquad \text{(transformation of currents).}$$

$$\Rightarrow I_p = \frac{1}{R}\left(\frac{N_s}{N_p}\right)^2 V_p. \Rightarrow R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R.$$

Here $R_{eq}$ is the value of the load resistance as "seen" by the generator.

For maximum transfer of energy from an emf device to a resistive load, the resistance of the emf device must equal the resistance of the load. For ac circuits, for the same to be true, the *impedance* (rather than just the resistance) of the generator must equal that of the load.