Physics 1

Chapter 13 Oscillations

1. Oscillations
2. Simple Harmonic Motion
   - Velocity of SHM
   - Acceleration of SHM
3. The Force Law for SHM
4. Energy in SHM
5. An Angular Simple Harmonic Oscillator
6. Pendulums
   - The Simple Pendulum
   - The Physical Pendulum
   - Measuring “g”
7. SHM & Uniform Circular Motion
8. Damped SHM
9. Forced Oscillations & Resonance

Oscillations

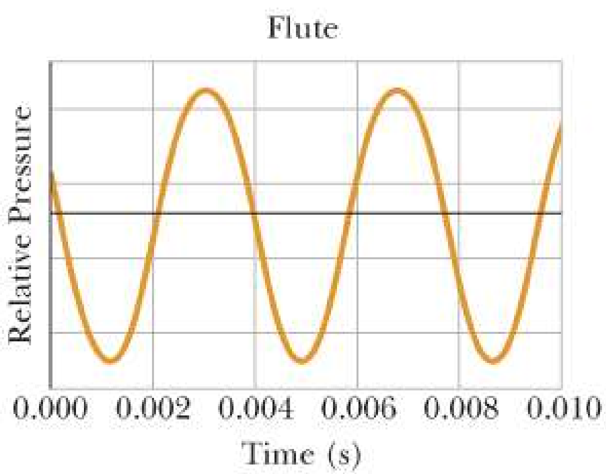
- Oscillations - motions that repeat themselves.

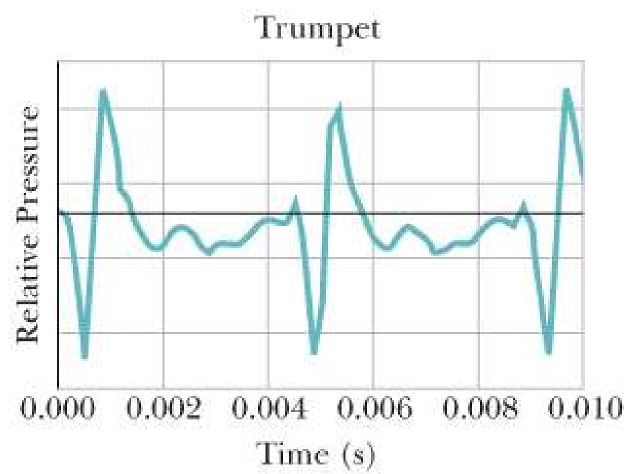
Oscillation occurs when a system is disturbed from a position of stable equilibrium.

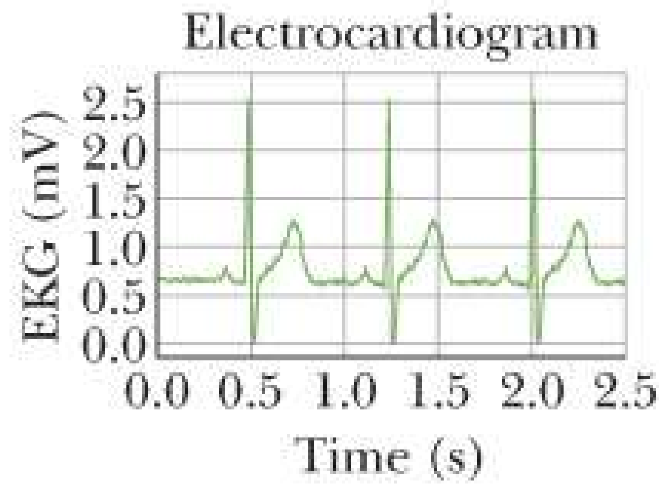
  - Clock pendulums swing
  - Boats bob up and down
  - Guitar strings vibrate
  - Diaphragms in speakers
  - Quartz crystals in watches
  - Air molecules
  - Electrons
  - Etc.

Oscillations

- <u>Oscillations</u> - motions that repeat themselves.





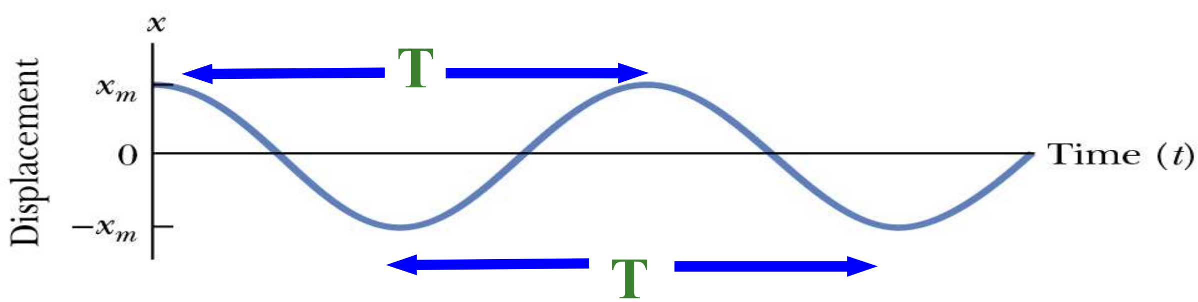


Simple Harmonic Motion

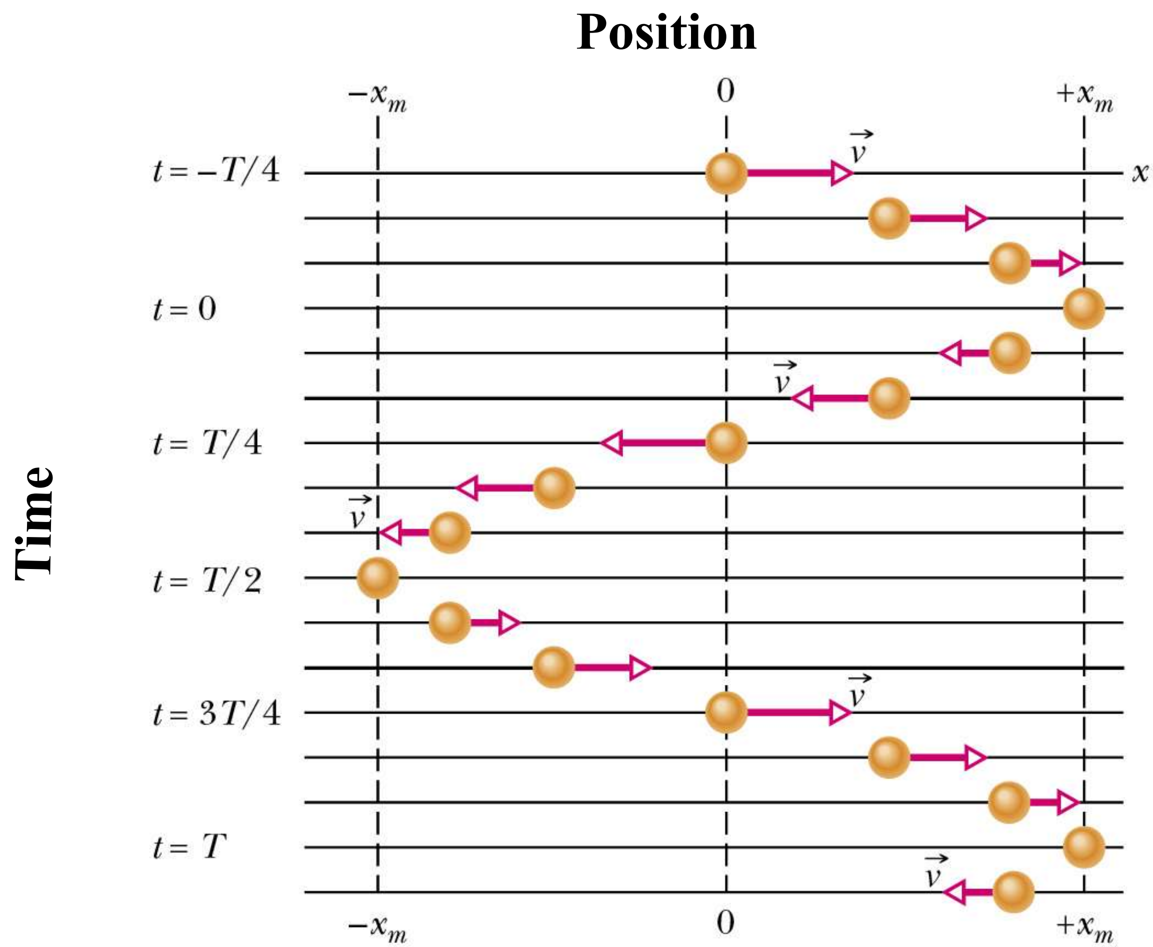
- *Harmonic Motion* - repeats itself at regular intervals (periodic).

- *Frequency* - # of oscillations per second
- 1 oscillation / s = 1 hertz (Hz)
-
- *Period* - time for one complete oscillation (one cycle)

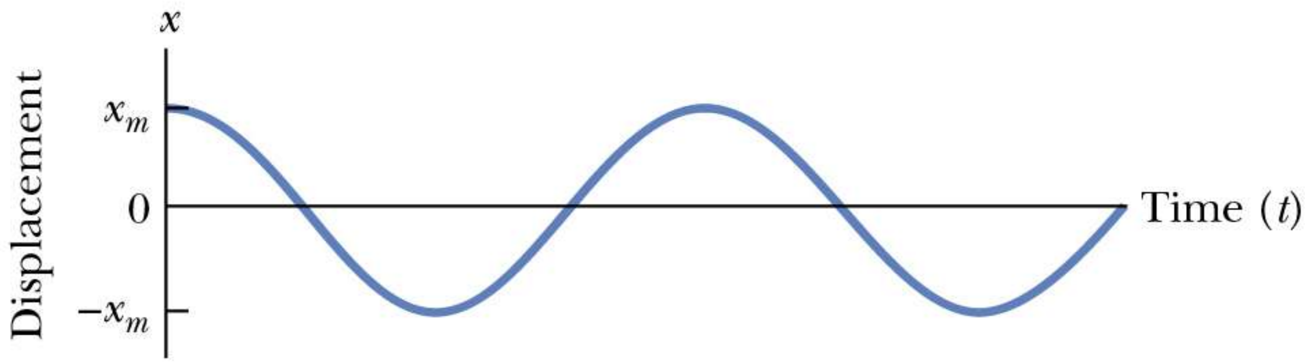
$$T = \frac{1}{f}$$

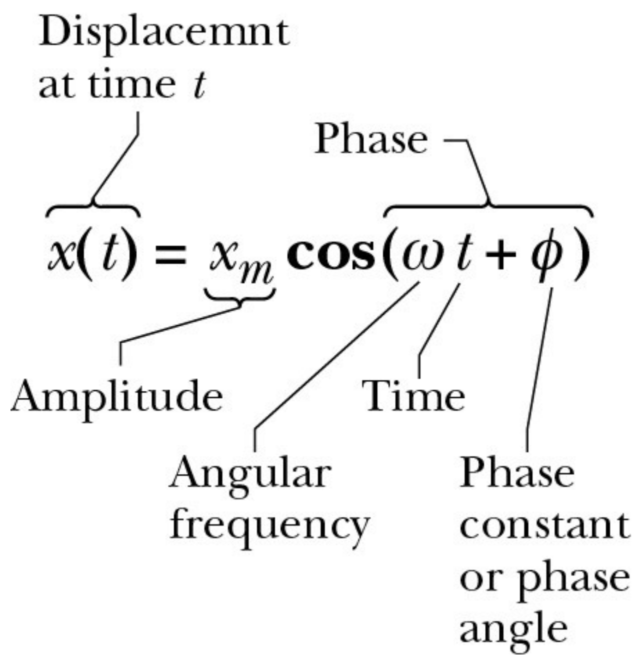


Simple Harmonic Motion



Simple Harmonic Motion





$$\omega = 2\pi f$$

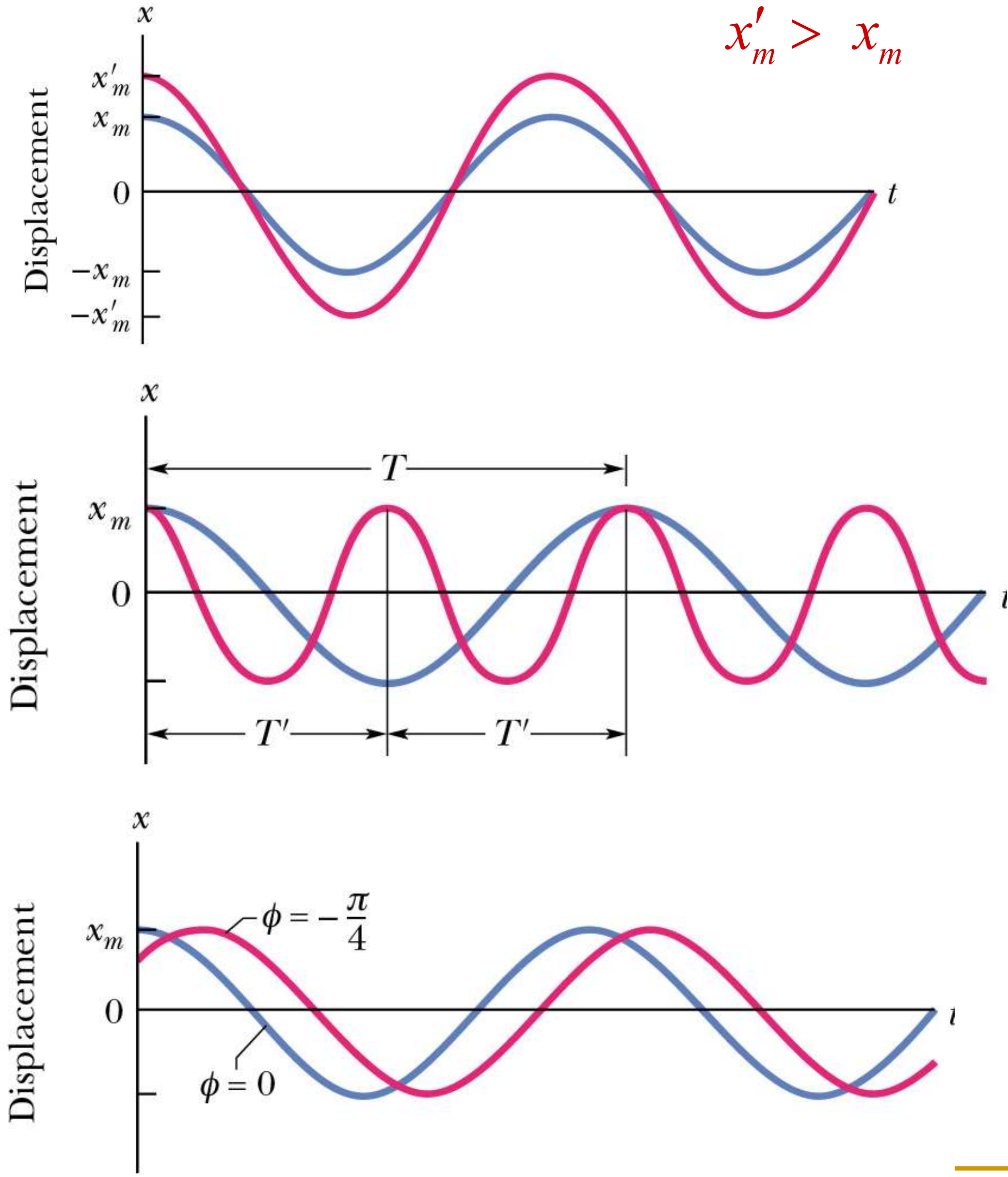
$$\text{radians}/s = \left(\text{radians}/\text{cycle}\right)\left(\text{cycles}/s\right)$$

$$T = \frac{1}{f}$$

$$\omega T = 2\pi$$

Angles are in radians.

Amplitude, Frequency & Phase



$$x(t) = x_m \cos(\omega t)$$

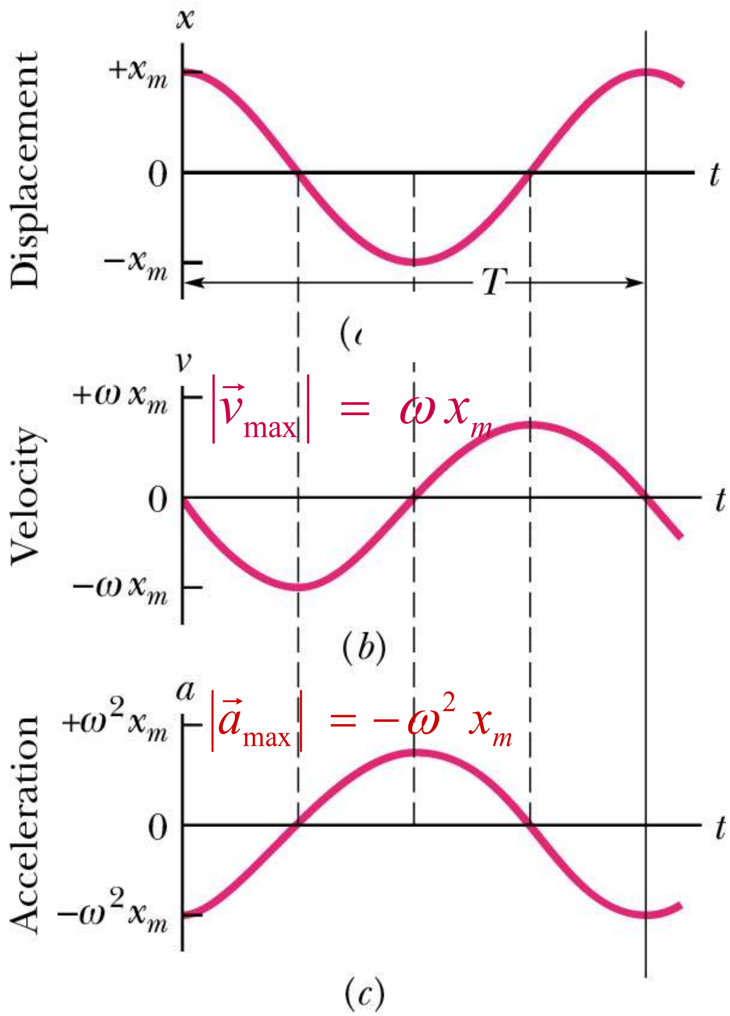
$$x(t) = x'_m \cos(\omega t)$$

$$x(t) = x_m \cos(2\omega t)$$

The frequency of SHM is **independent** of the amplitude.

$$x(t) = x_m \cos\left(\omega t - \pi/4\right)$$

Velocity & Acceleration of SHM



$$x(t) = x_m \cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt}$$

$$v(t) = -\omega\, x_m \sin(\omega t + \phi)$$

The phase of v(t) is shifted ¼ period relative to x(t),

$$a(t) = \frac{dv}{dt}$$

$$a(t) = -\omega^2\, x_m \cos(\omega t + \phi)$$

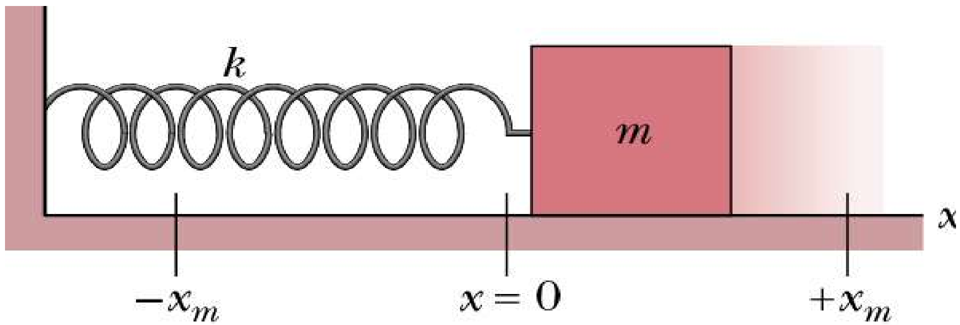
$$a(t) = -\omega^2\, x(t)$$

In SHM, a(t) is proportional to x(t) but opposite in sign.

The Force Law for SHM

- ***Simple Harmonic Motion*** is the motion executed by a particle of mass *m* subject to a force proportional to the displacement of the particle but opposite in sign.

Hooke's Law: $F(t) = -k\,x(t)$



"Linear Oscillator": F ~ - x

The Differential Equation that Describes SHM

$$F(t) = -k\,x(t)$$

- *Simple Harmonic Motion* is the motion executed by a particle of mass *m* subject to a force proportional to the displacement of the particle but opposite in sign. Hooke's Law!

Newton's 2nd Law: $F = m\,a$

$$m\frac{d^2x}{dt^2} = -k\,x$$

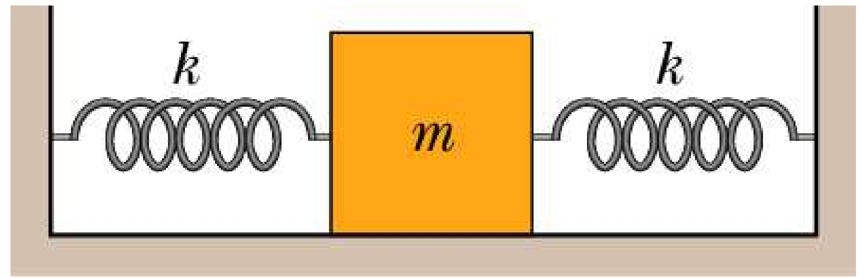
$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \text{where} \quad \omega^2 = \frac{k}{m}$$

The **general solution** of this **differential equation** is:

$$x(t) = x_m \cos(\omega t + \phi)$$

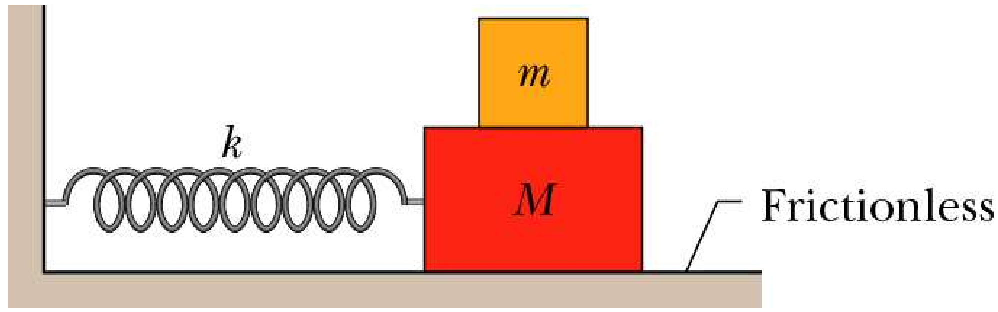
What is the frequency?

k = 7580 N/m
m = 0.245 kg
f = ?

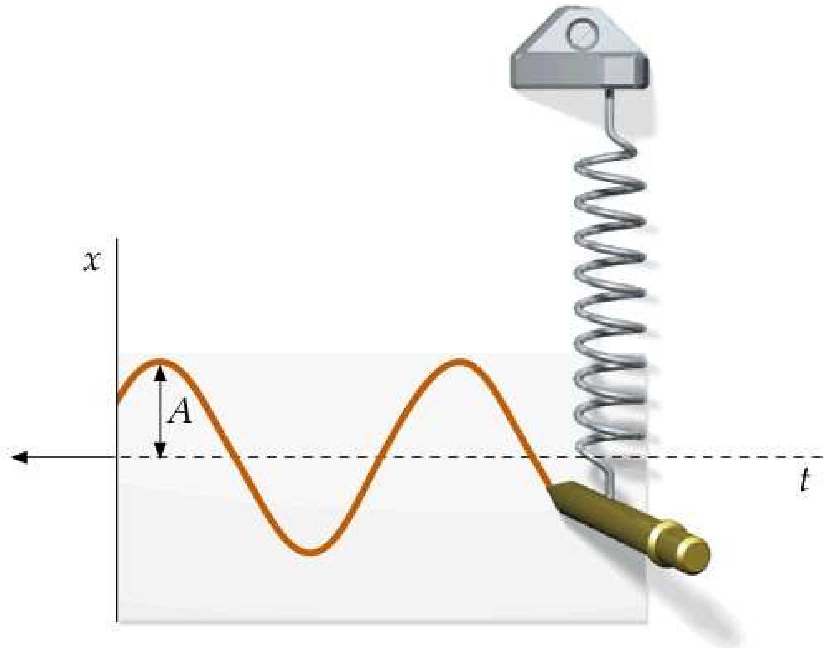


x_m without m falling off?

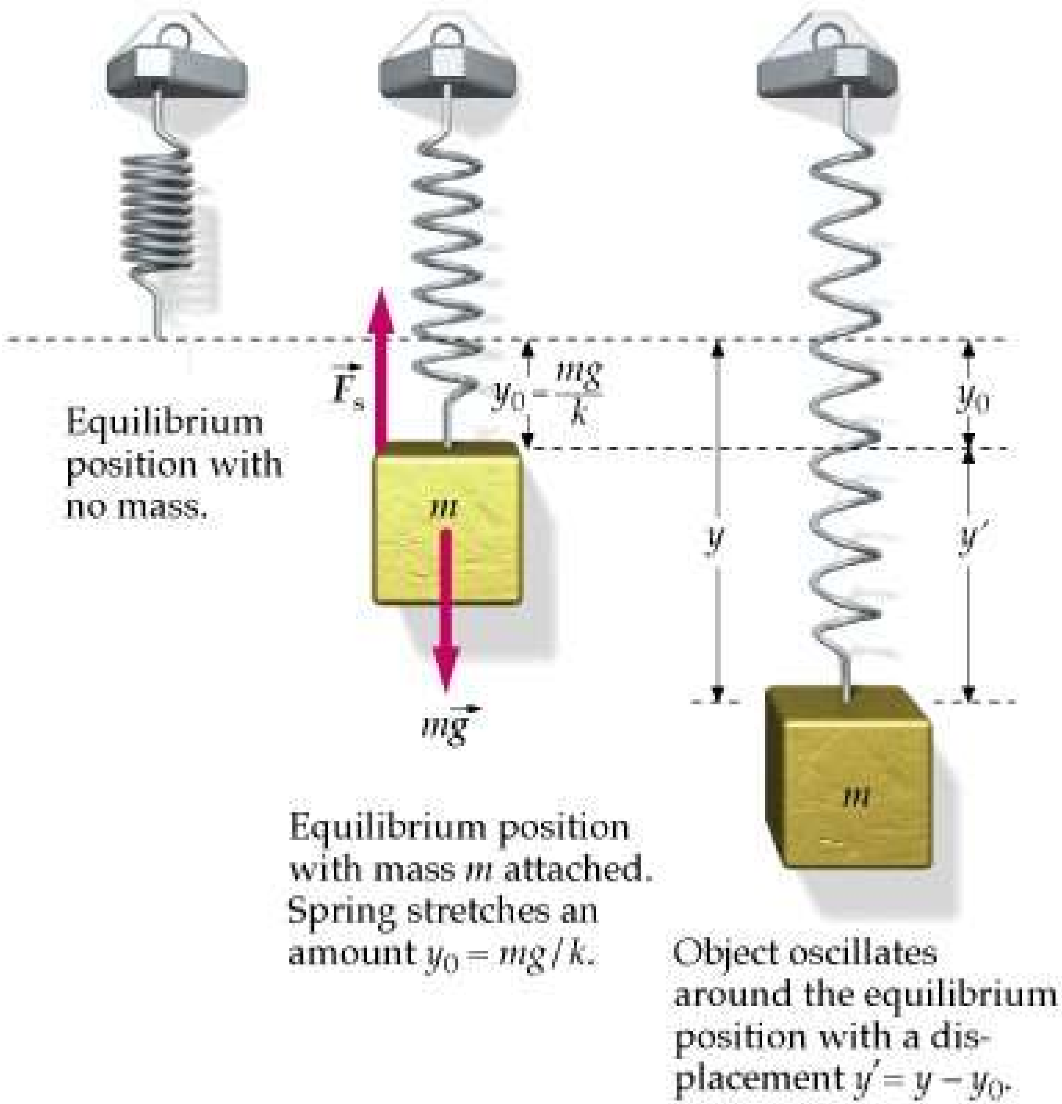
m = 1.0 kg
M = 10 kg
k = 200 N/m
μ_s = 0.40
Maximum x_m without slipping



Simple Harmonic Motion



Vertical Spring Oscillations



Energy in Simple Harmonic Motion

$$K = \tfrac{1}{2} m v^2 \qquad U = \tfrac{1}{2} k x^2$$

$$E = K + U$$

$$K(t) = \tfrac{1}{2} m \left(-\omega\ x_m \sin(\omega t + \phi)\right)^2$$

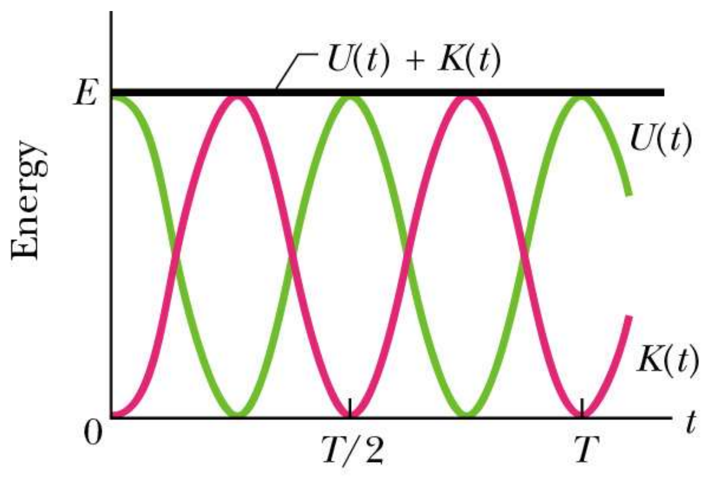
$$k = m\omega^2$$

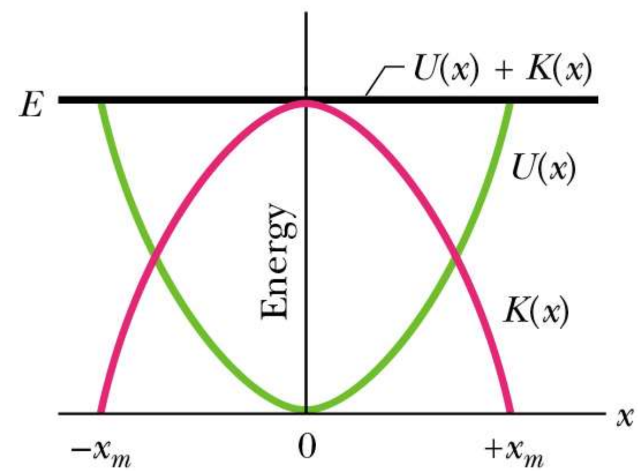
$$K(t) = \tfrac{1}{2} k\, x_m^2 \sin^2(\omega t + \phi)$$

$$U(t) = \tfrac{1}{2} k \left(x_m \cos(\omega t + \phi)\right)^2$$

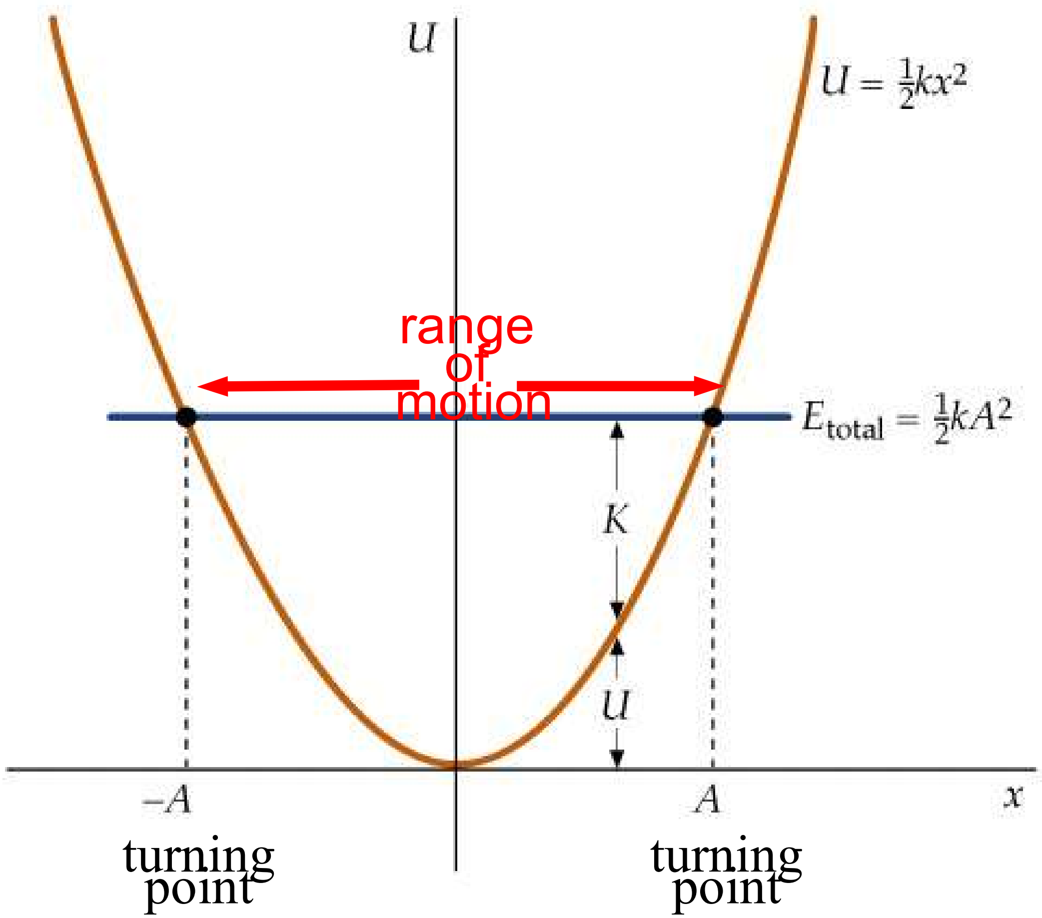
$$U(t) = \tfrac{1}{2} k\ x_m^2 \cos^2(\omega t + \phi)$$

$$\therefore\ E = \tfrac{1}{2} k\, x_m^2 = constant$$





Energy in Simple

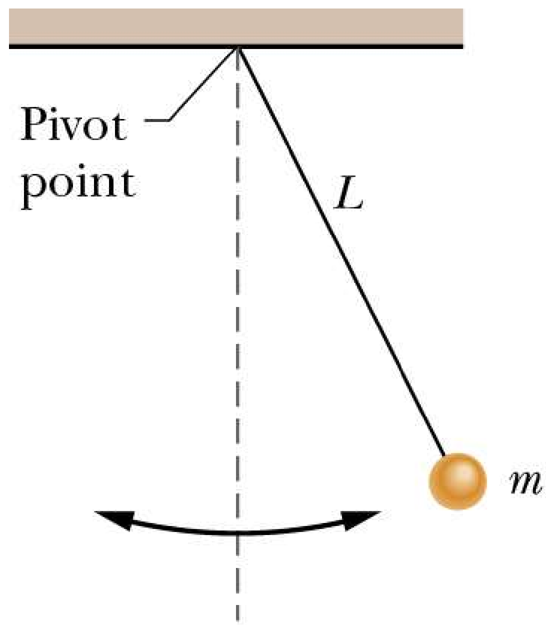


$$U = \tfrac{1}{2} k x^2$$

$$E = \tfrac{1}{2} k x_m^2 = \text{constant}$$

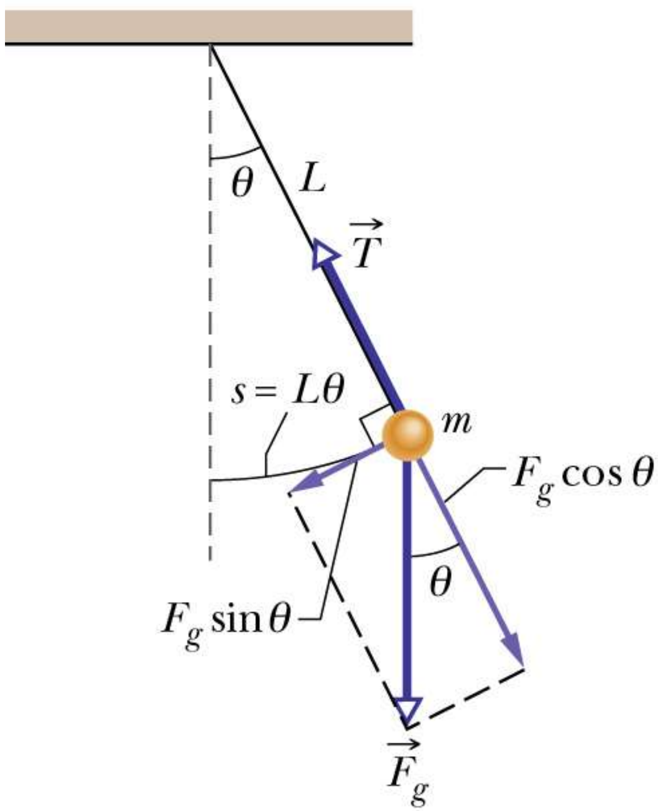
Gravitational Pendulum

Simple Pendulum: a bob of mass **m** hung on an unstretchable massless string of length **L**.



Simple Pendulum

Simple Pendulum: a bob of mass **m** hung on an unstretchable massless string of length **L**.



m

SHM for small θ

$$\tau = -L F_g \sin\theta \approx -L F_g \theta$$

$$\tau = I\alpha$$

$$\alpha = -\frac{m g L}{I}\theta \qquad I = mL^2$$

acceleration ~ - displacement

SHM

$$a(t) = -\omega^2 x(t)$$

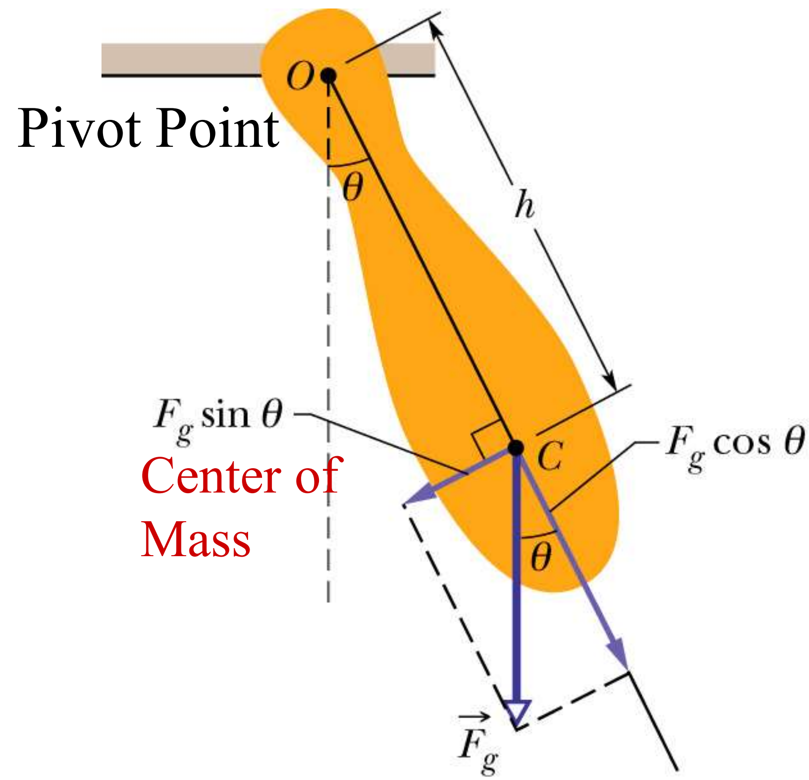
$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

A pendulum leaving a trail of ink:

Physical Pendulum

A rigid body pivoted about a point other than its center of mass (com). <u>SHM for small θ</u>



quick method to measure g

$$\tau = -hF_g \sin\theta \approx -hF_g\,\theta$$

$$\tau = I\alpha$$

$$\alpha = -\frac{mgh}{I}\theta$$

acceleration ~ - displacement

SHM

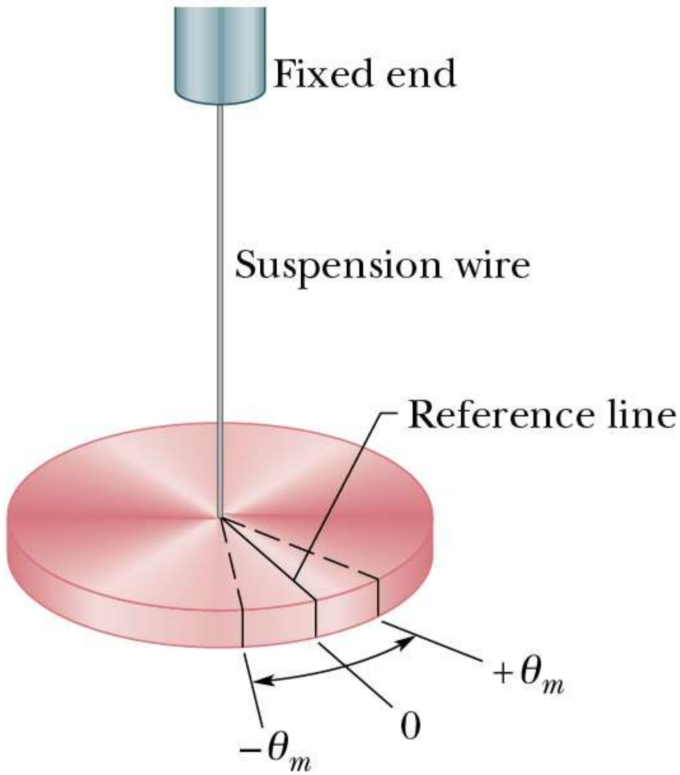
$$a(t) = -\omega^2 x(t)$$

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Angular Simple Harmonic Oscillator

Torsion Pendulum: $\tau \sim \theta$



$$\tau = \kappa\theta$$ Hooke's Law

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} = \kappa\theta$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

Spring: $m \Leftrightarrow I$

$k \Leftrightarrow \kappa$

Simple Harmonic Motion

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad T = 2\pi\sqrt{\frac{L}{g}} \qquad T = 2\pi\sqrt{\frac{I}{mgh}} \qquad T = 2\pi\sqrt{\frac{I}{\kappa}}$$

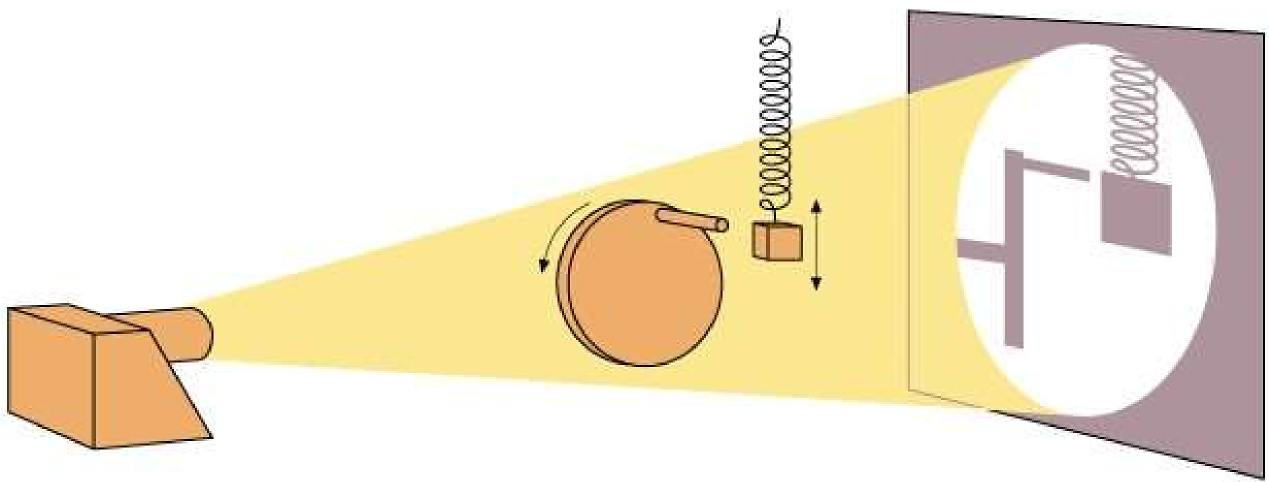
<u>Any Oscillating System</u>:

"inertia" versus "springiness"

$$T = 2\pi\sqrt{\frac{\text{inertia}}{\text{springiness}}}$$

SHM & Uniform Circular Motion

The projection of a point moving in uniform circular motion on a diameter of the circle in which the motion occurs executes SHM.



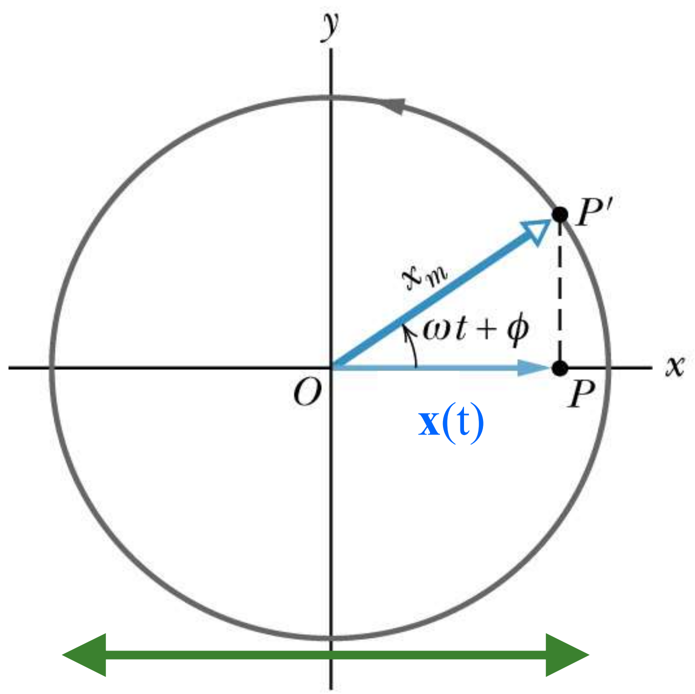
The execution of uniform circular motion describes SHM.

SHM & Uniform Circular Motion

The reference point P' moves on a circle of radius $\mathbf{x_m}$.
The projection of $\mathbf{x_m}$ on a diameter of the circle executes SHM.

radius = $\mathbf{x}_\mathrm{m}$

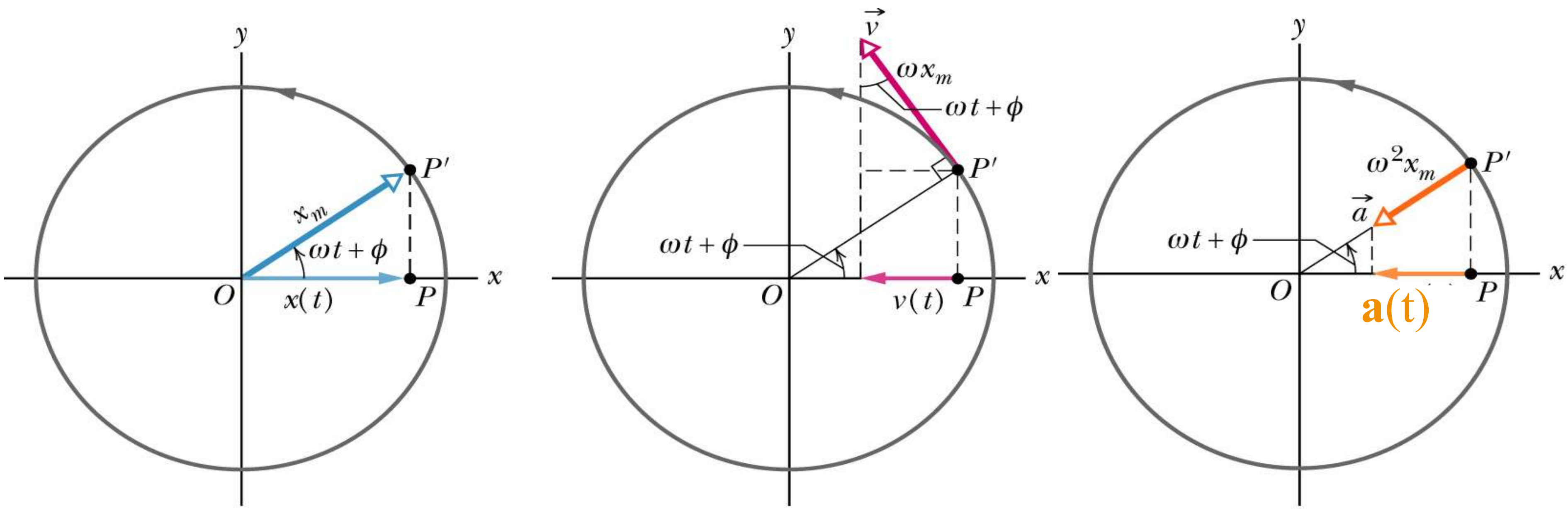
$angle = \omega t + \phi$



$$x(t) = x_m \cos(\omega t + \phi)$$

SHM & Uniform Circular Motion

The reference point P’ moves on a circle of radius $\mathbf{x_m}$.
The projection of $\mathbf{x_m}$ on a diameter of the circle executes <u>SHM</u>.



$$x(t) = x_m \cos(\omega t + \phi)$$

$$\text{radius} = \mathbf{x}_\mathrm{m}$$

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

$$|\vec{v}| = \omega x_m$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

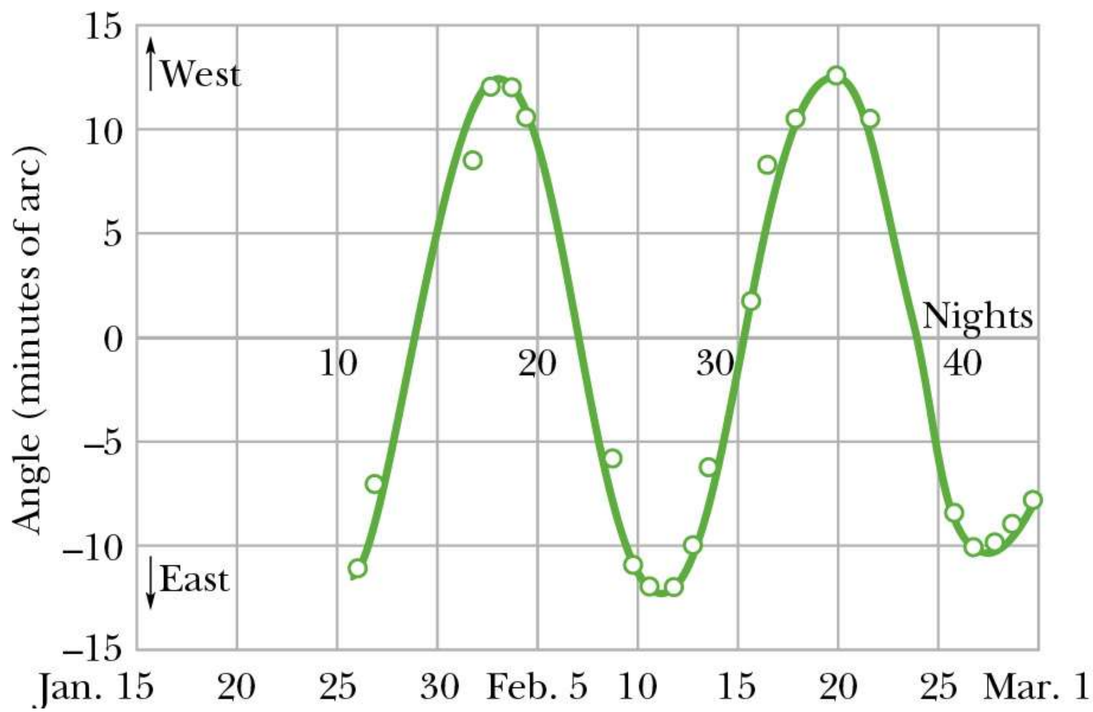
$$|\vec{a}| = -\omega^2 x_m$$

SHM & Uniform Circular Motion

The projection of a point moving in uniform circular motion on a diameter of the circle in which the motion occurs executes SHM.

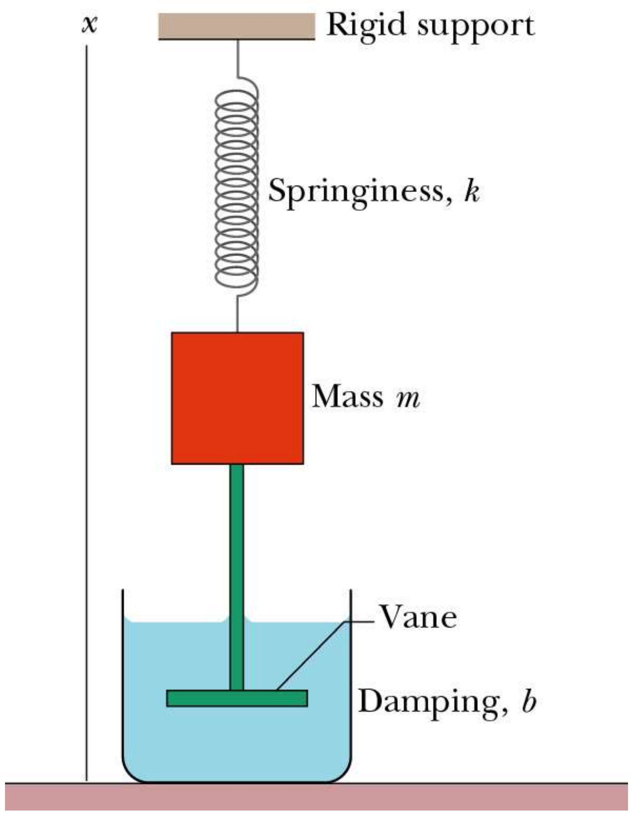
<u>Measurements of the angle between Callisto and Jupiter</u>:

Galileo (1610)



Damped SHM

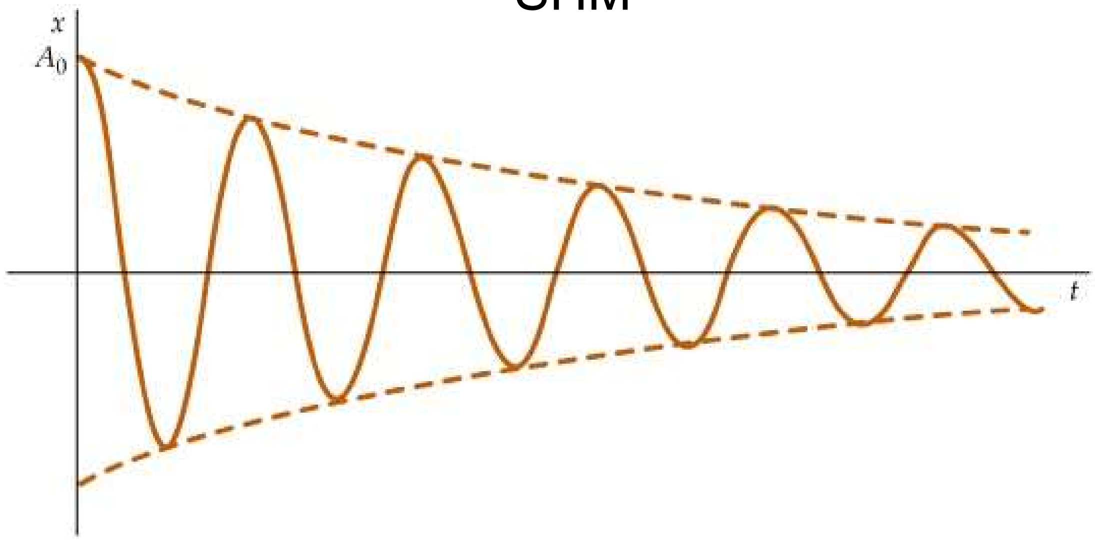
SHM in which each oscillation is reduced by an external force.



$$F = -k\,x$$

Restoring Force

SHM



$$F_D = -b\,v$$

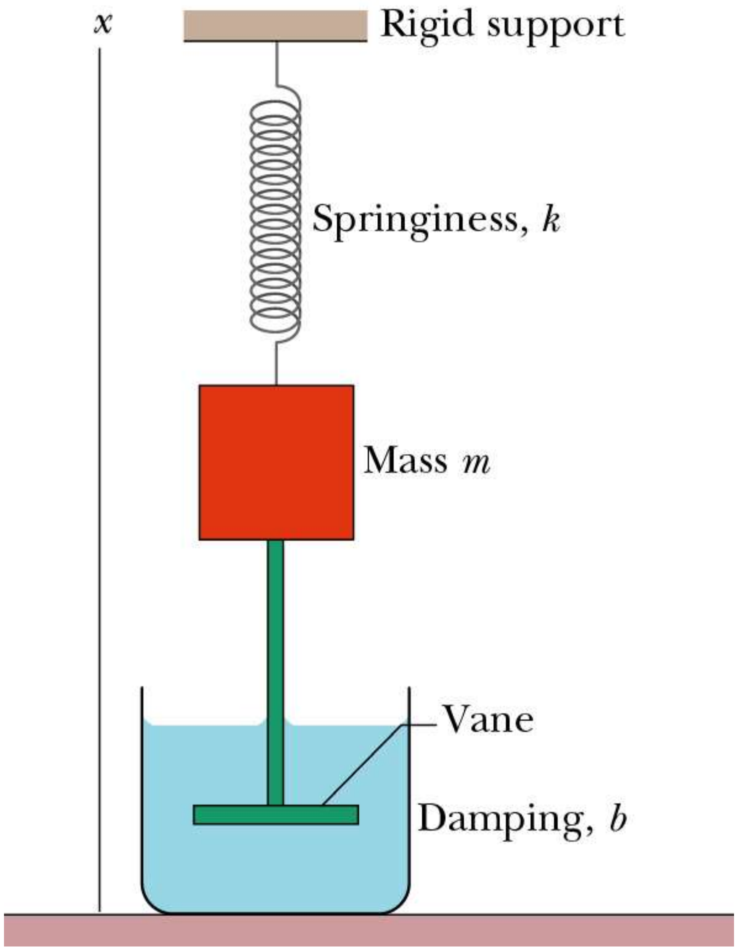
Damping Force

In opposite direction to velocity

Does negative work

Reduces the mechanical energy

Damped SHM



$$F_{net} = m\,a$$

$$-k\,x - b\,v = m\,a$$

$$-k\,x - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

<u>differential equation</u>

Damped Oscillations

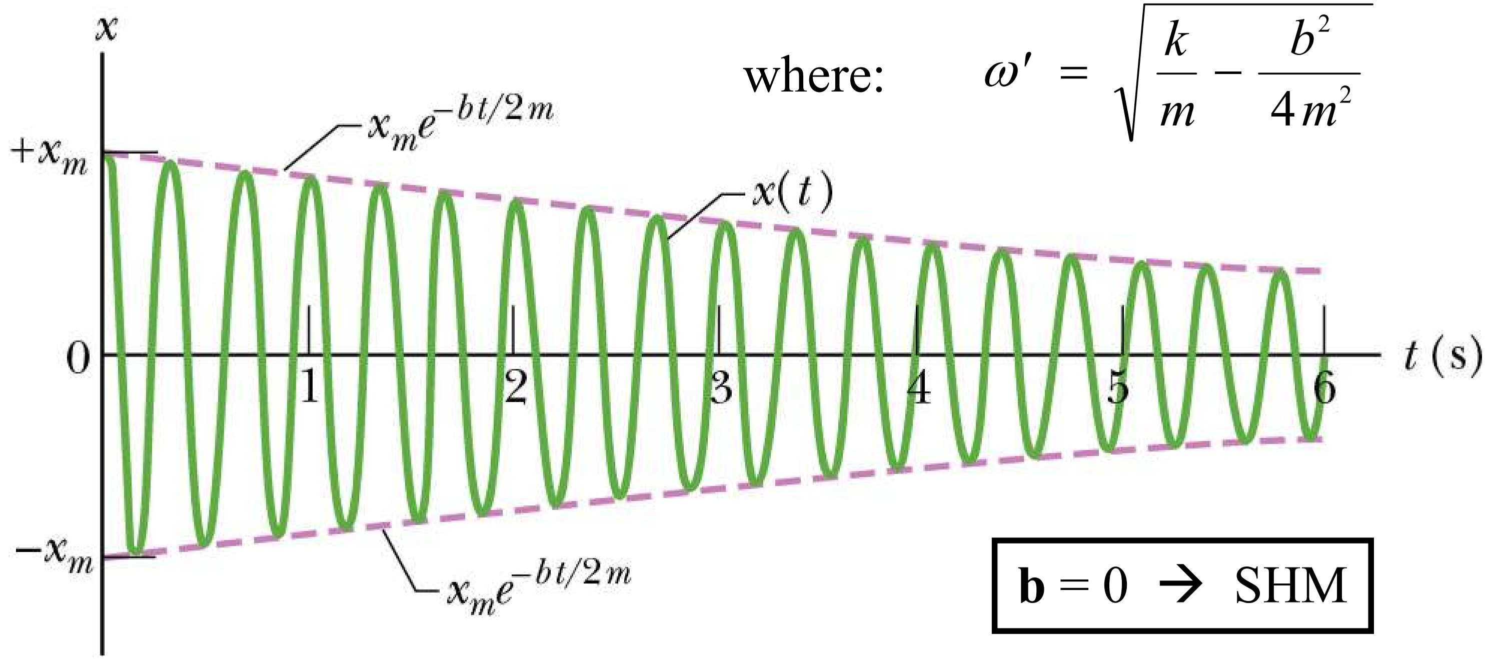
2nd Order Homogeneous Linear Differential Equation:

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

Solution of Differential Equation:

$$x(t) = x_m e^{-\frac{b}{2m}t}\cos(\omega' t + \phi)$$

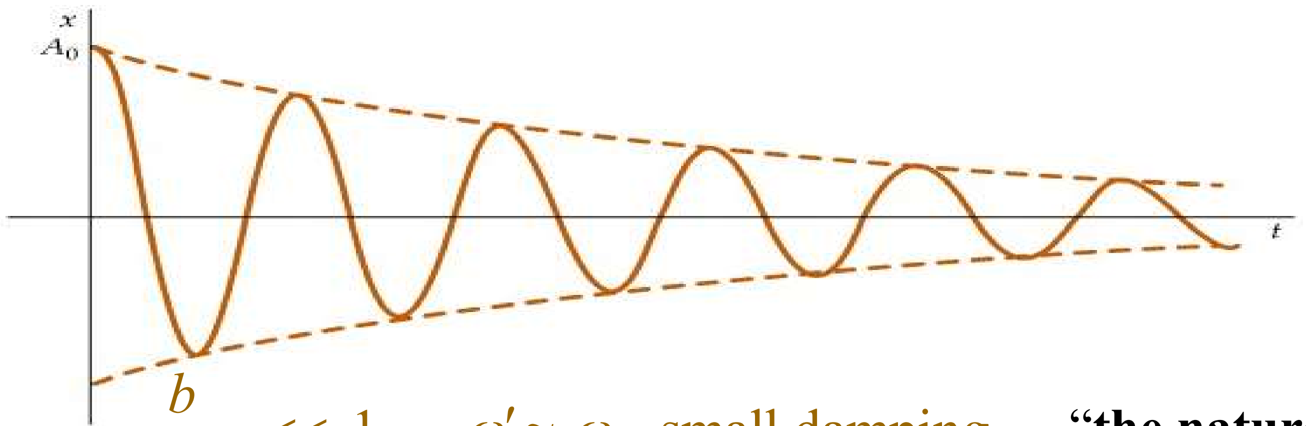
where: $$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$



b = 0 → SHM

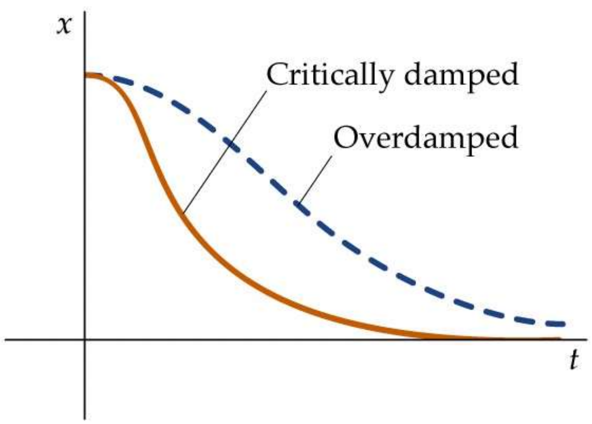
Damped Oscillations

$$x(t) = x_m e^{-\frac{b}{2m}t} \cos(\omega' t + \phi) \qquad \omega' = \omega\sqrt{1 - \left(\frac{b}{2m\omega}\right)^2}$$



$$\omega = \sqrt{\frac{k}{m}}$$

$\frac{b}{2m\omega} << 1 \quad \omega' \approx \omega$ small damping **"the natural frequency"**

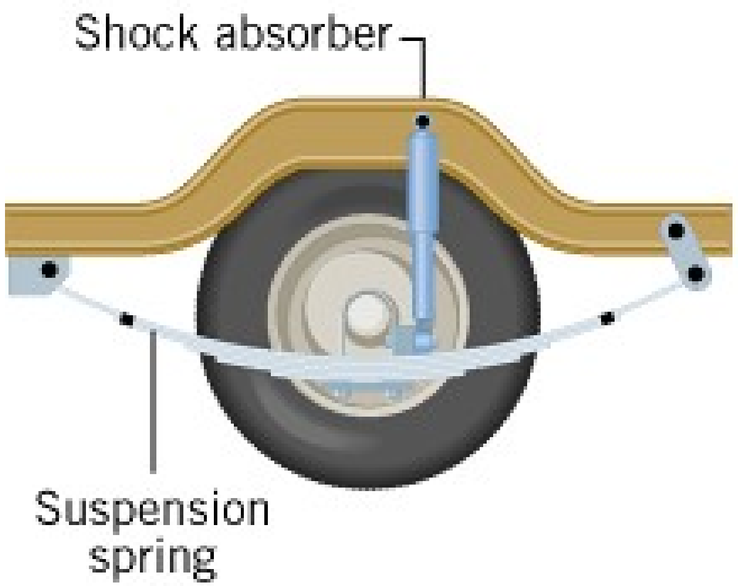


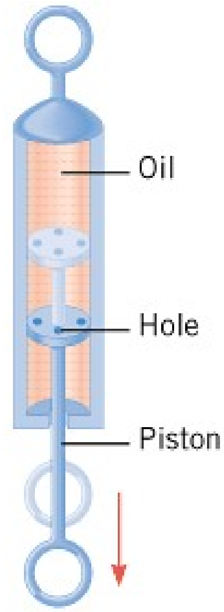
$\frac{b}{2m\omega} \approx 1 \quad \omega' \approx 0$ "*critically damped*"

$\frac{b}{2m\omega} > 1 \quad \omega^2 < 0$ "*overdamped*"

Exponential solution to the DE

Auto Shock Absorbers

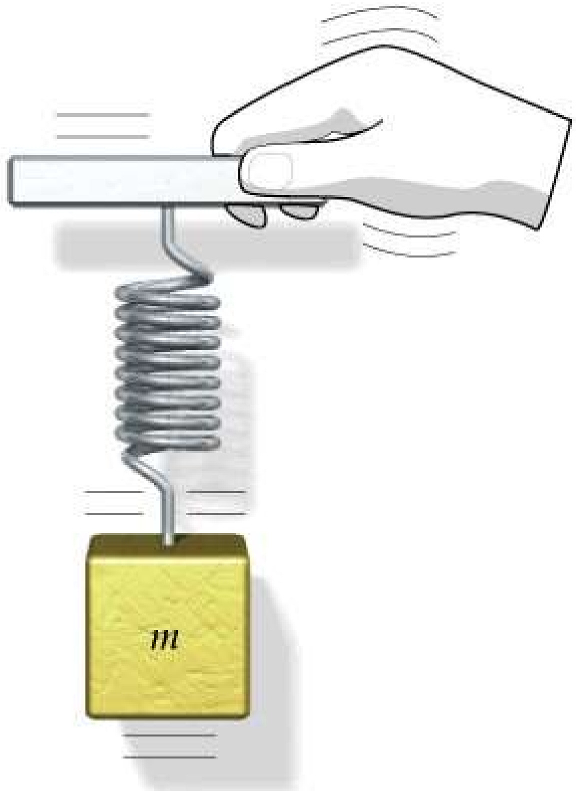




Typical automobile shock absorbers are designed to produce slightly under-damped motion

Forced Oscillations

Each oscillation is driven by an external force to maintain motion in the presence of damping:



$$F_0 \cos(\omega_d t)$$

ω_d = **driving frequency**

Forced Oscillations

Each oscillation is driven by an external force to maintain motion in the presence of damping.

$$F_{net} = m\,a$$

$$-k\,x - b\,v + F_0 \cos\left(\omega_d\, t\right) = m\,a$$

2nd Order Inhomogeneous Linear Differential Equation:

$$m\frac{d^2x}{dt^2} + k\,x + m\,\omega^2 \frac{dx}{dt} = F_0 \cos\left(\omega_d\, t\right)$$

$$\omega = \sqrt{\frac{k}{m}}$$

"the natural frequency"

Forced Oscillations & Resonance

2nd Order Homogeneous Linear Differential Equation:

$$m\frac{d^2x}{dt^2} + k\,x + m\omega^2\frac{dx}{dt} = F_0\cos(\omega_d\,t)$$

Steady-State Solution of Differential Equation:

$$x(t) = x_m\cos(\omega\,t + \delta)$$

where:

$$x_m = \frac{F_0}{\sqrt{m^2\left(\omega^2 - \omega_d^{\,2}\right)^2 + b^2\,\omega_d^{\,2}}}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\tan\delta = \frac{b\,\omega_d}{m\left(\omega^2 - \omega_d^{\,2}\right)}$$

ω = natural frequency

ω_d = driving frequency

Forced Oscillations & Resonance

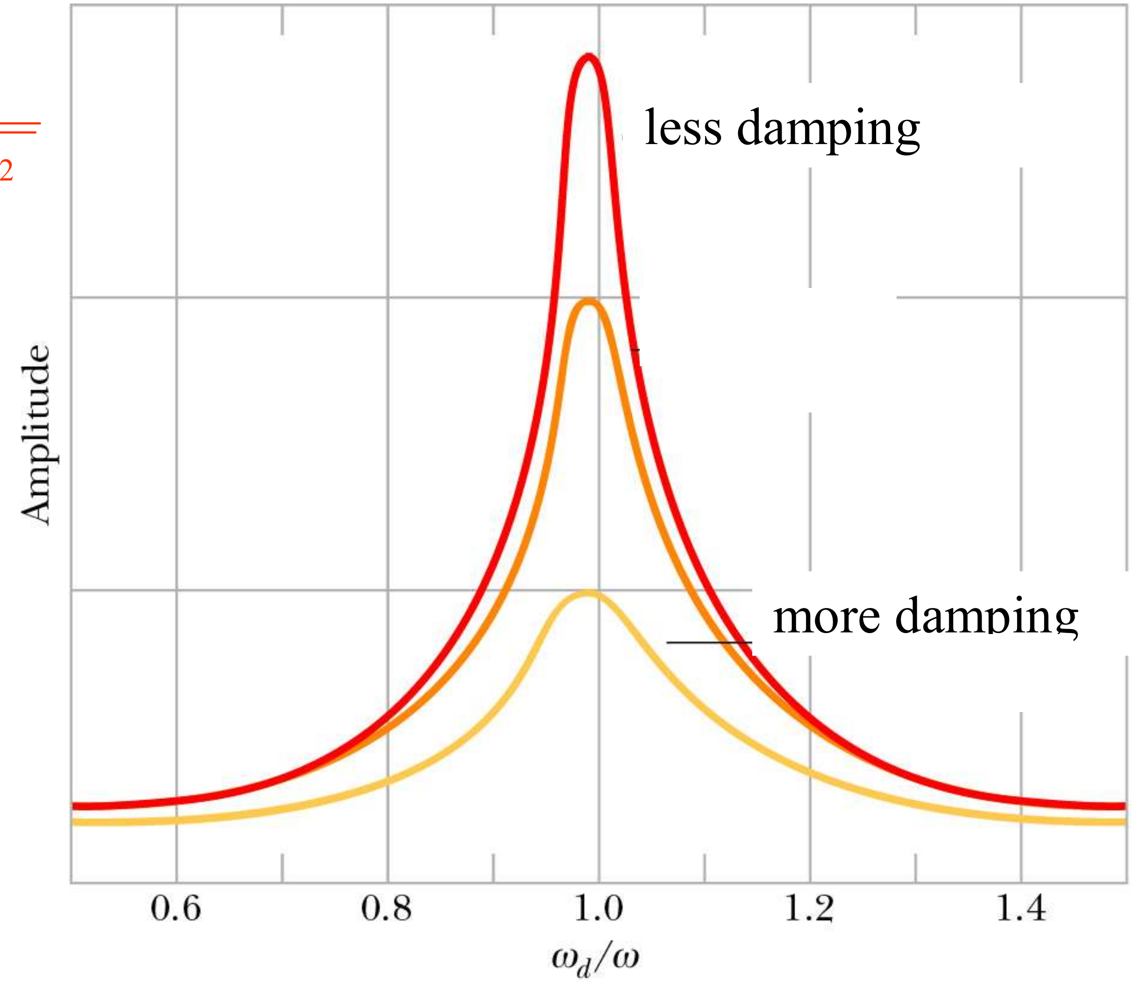
The natural frequency, ω, is the frequency of oscillation when there is no external driving force or damping.

$$x_m = \frac{F_0}{\sqrt{m^2\left(\omega^2 - \omega_d^{\ 2}\right)^2 + b^2\,\omega_d^{\ 2}}}$$

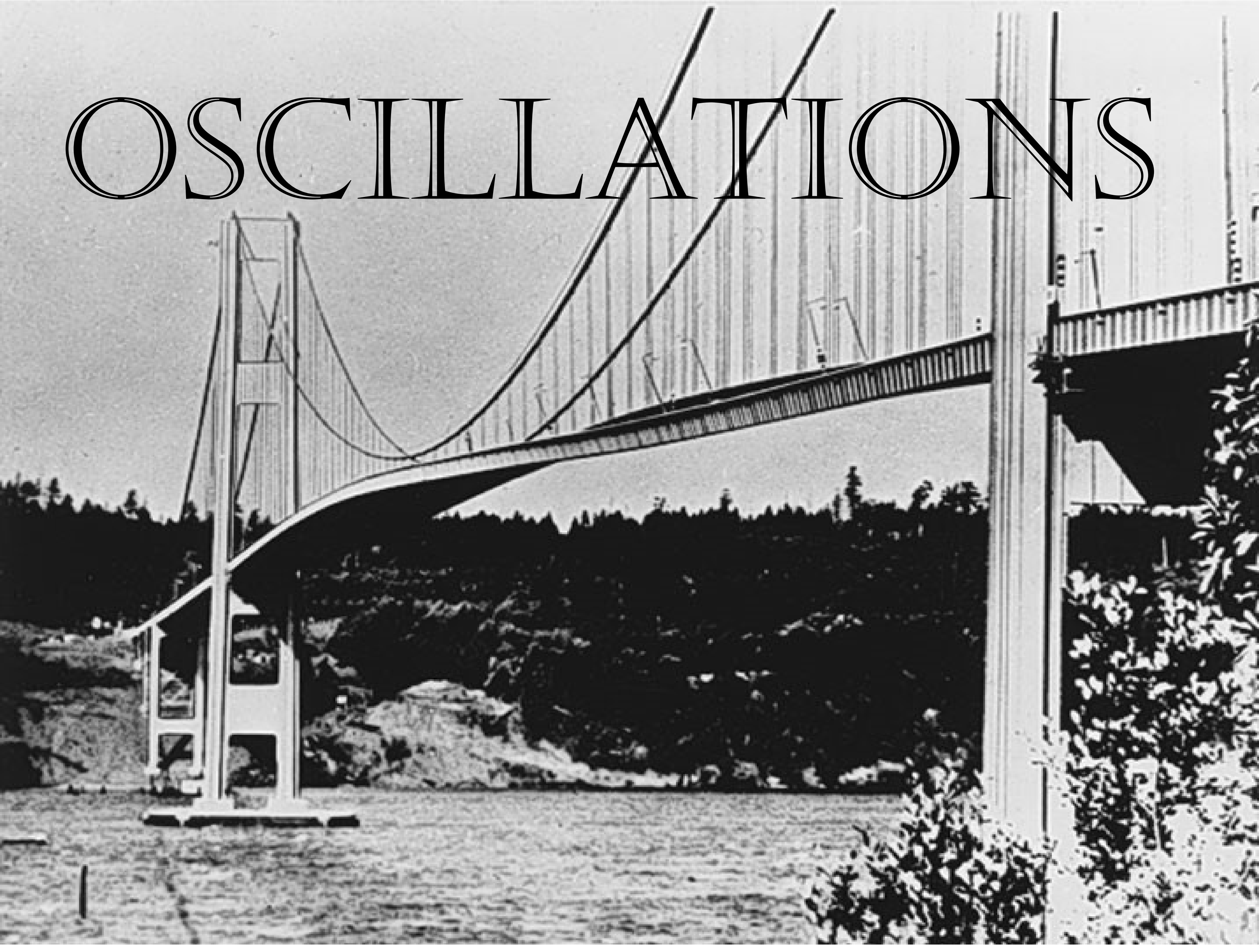
$$\omega = \sqrt{\frac{k}{m}}$$

ω = **natural** frequency

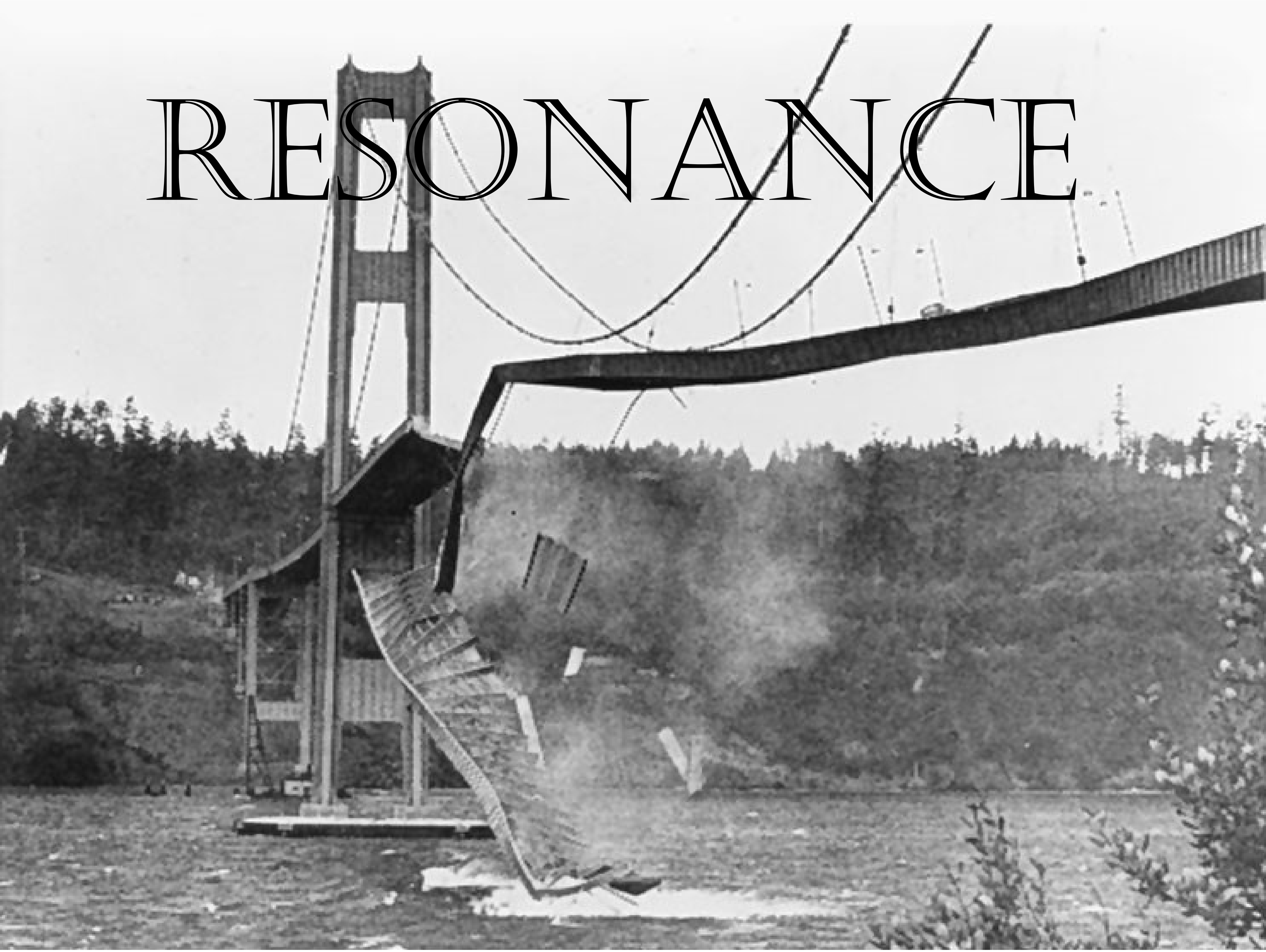
ω_d = **driving** frequency



When $\omega = \omega_d$ resonance occurs!


OSCILLATIONS

RESONANCE



Stop the SHM caused by winds on a high-rise building

400 ton weight mounted on a spring on a high floor of the Citicorp building in New York.

The weight is forced to oscillate at the same frequency as the building but 180 degrees out of phase.

Forced Oscillations & Resonance

Mechanical Systems

$$m\frac{d^2x}{dt^2} + k\,x + m\omega^2\frac{dx}{dt} = F_0\cos(\omega_d\,t)$$

e.g. the forced motion of a mass on a spring

Electrical Systems

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = \mathcal{E}_m\sin\omega_d t$$

e.g. the charge on a capacitor in an LRC circuit