Chapter 8 Potential Energy & Conservation of Energy

Potential Energy

Potential Energy is energy that can be associated with the configuration of a system of objects that exert forces on one another.

- *Gravitational Potential Energy*
- *Elastic Potential Energy*

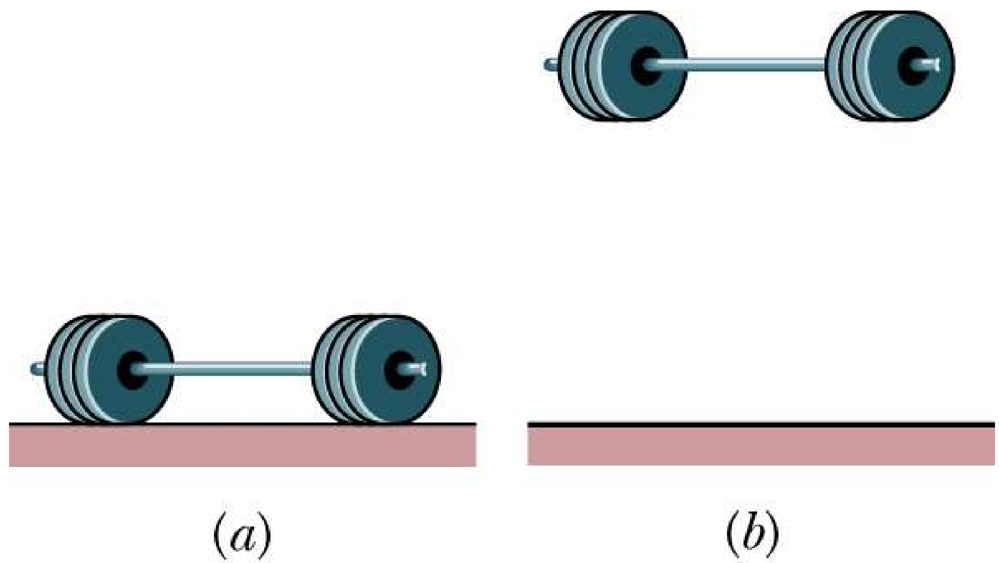
Chapter 7 - Kinetic Energy - state of motion of objects in a system.

$$Kinetic\ Energy \equiv \tfrac{1}{2} m v^2$$

Gravitational Potential Energy

Gravitational potential energy – energy in a set of separated objects which attract one another via the gravitational force.

the system = earth + barbell

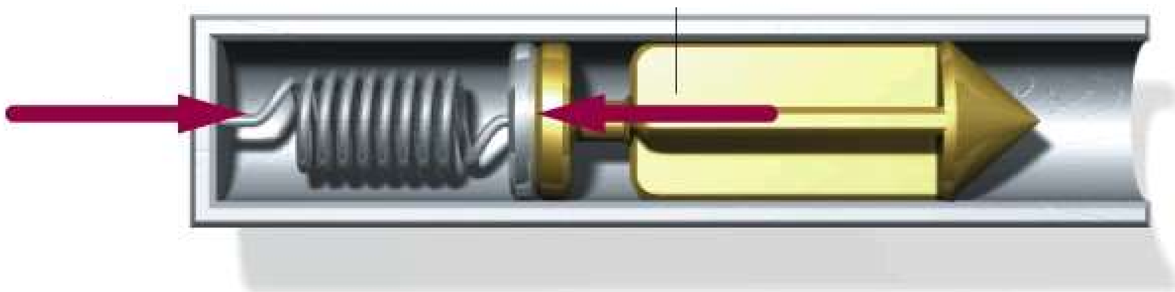


Increased P.E.

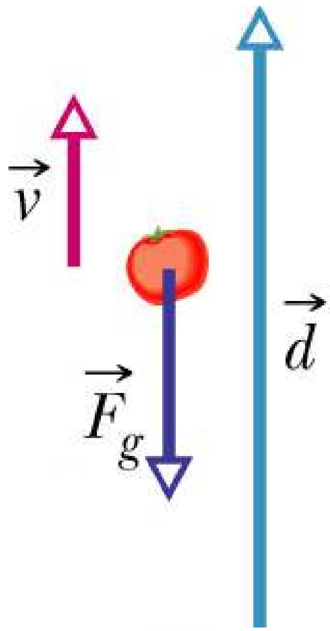
Elastic Potential Energy

Elastic potential energy – energy in a compressed or stretched spring-like object.

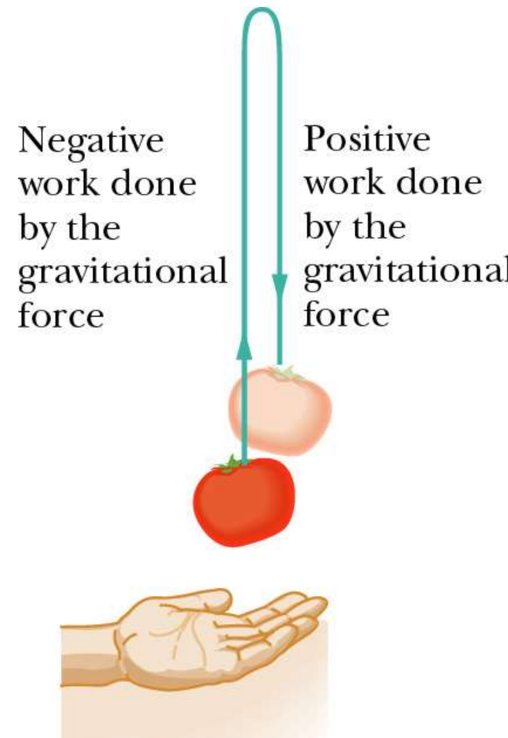
e.g.: *Elastic potential energy* stored in a dart gun.



The *change in the gravitational potential energy, U,* is <u>defined</u> to equal the negative of the work done <u>by the gravitational force</u>, W_g.

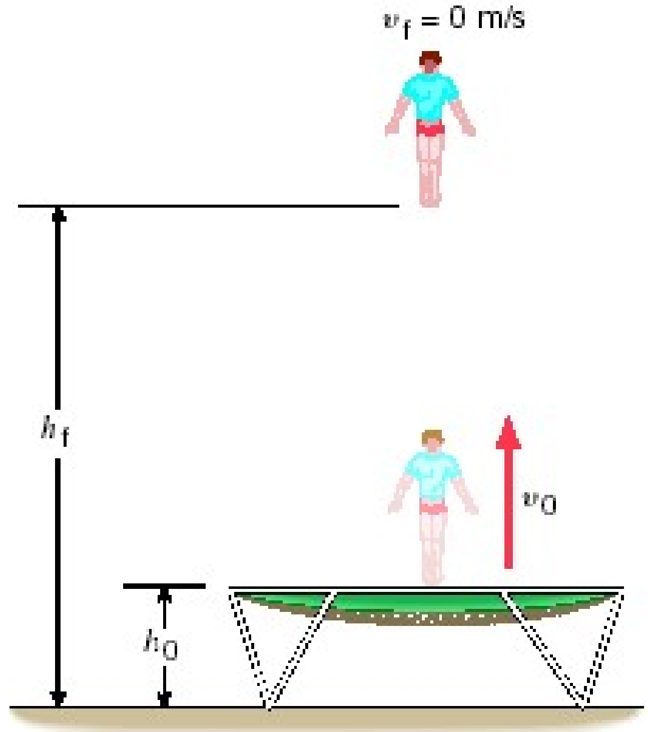


$$\Delta U = -W_g$$



Work done by Gravity

leaves at 1.20 m
max h = 4.80 m
what is v_0



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Conservative & Nonconservative Forces

Conservative Force Example

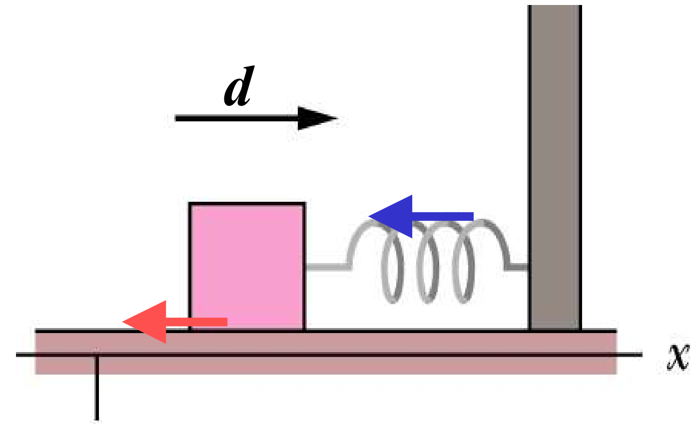
- Spring Force

$$W_1 = -W_2$$

An important concept!

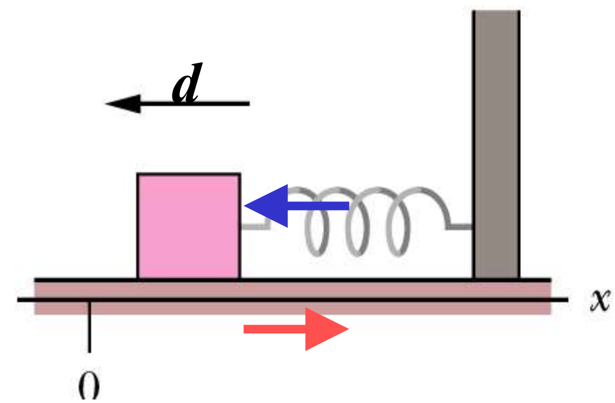
Nonconservative Force Example

- Friction Force



W_1 = *Negative Work done by the spring*

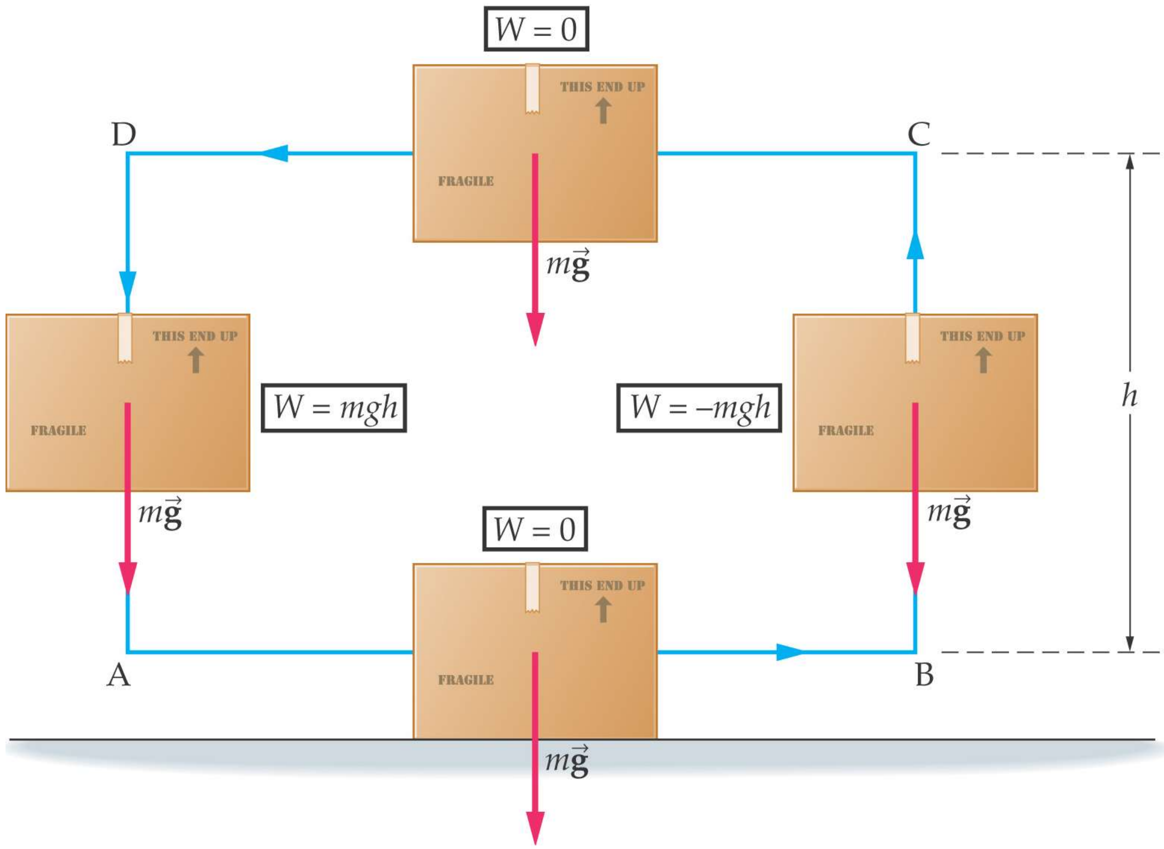
Negative Work done by friction



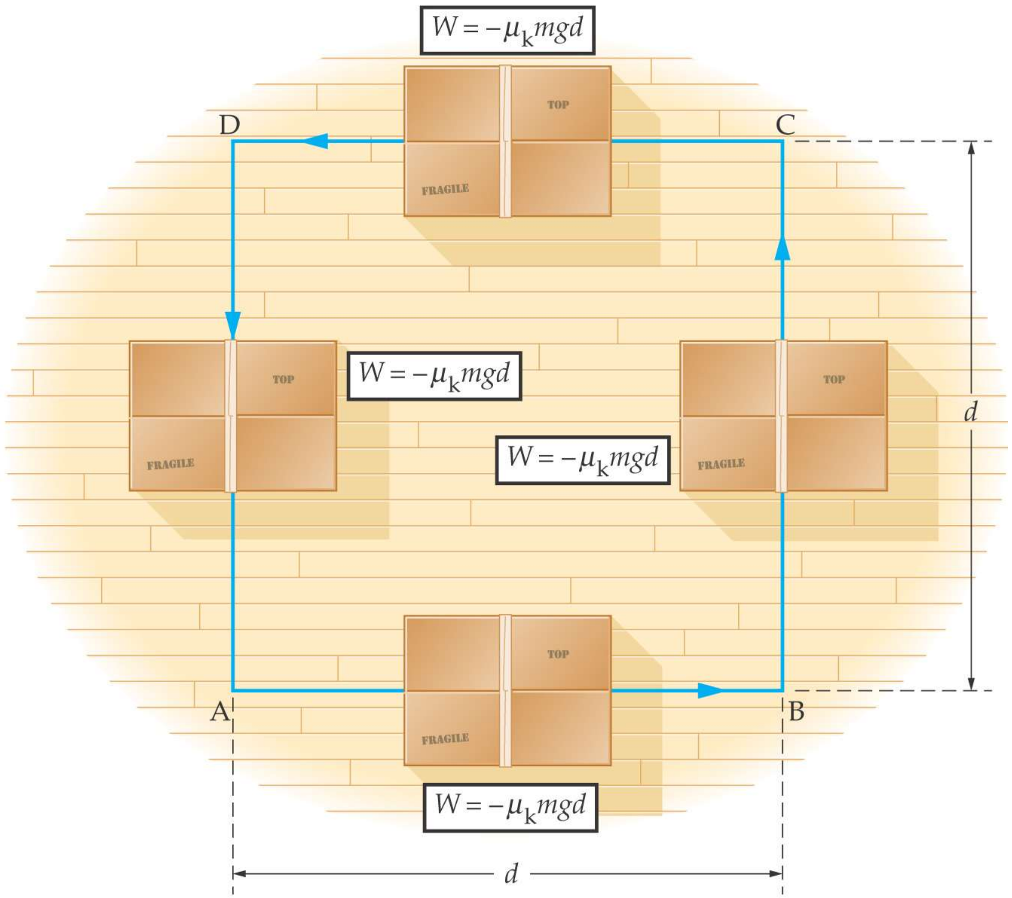
W_2 = *Positive Work done by the spring*

Negative Work done by friction

Work Done by Gravity on a Closed Path



Work Done by Friction on a Closed Path



Conservative Forces

- Work done by the gravitation force does not depend upon the choice of paths – a conservative force.

- **Definition of a Conservative Force**

Version 1 – A force is conservative when the work it does on a moving object is independent of the path between the objects initial and final position.

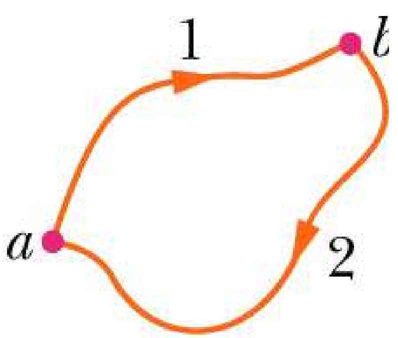
Version 2 – A force is conservative when it does no net work on an object moving around a closed path, starting and finishing at the same point

Path Independence of Conservative Forces

Consider a particle moving under the influence of a conservative force; then:

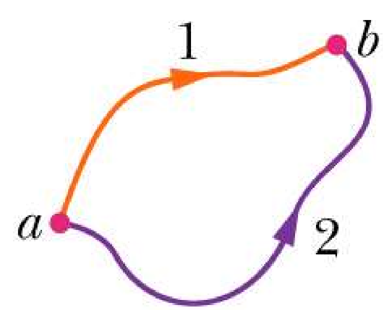
$$W_{ab} = - W_{ba}$$

Furthermore:



The net work done by a conservative force on a particle moving around every closed path is ***<u>zero</u>***:

$$W_{ab,1} + W_{ba,2} = 0$$



The work done by a conservative force on a particle moving between two points ***does not depend on the path*** taken by the particle.

$$W_{ab,1} = W_{ab,2}$$

Determining Potential Energy Values

The change in the potential energy is defined to equal the negative of the work done by the forces in the system:

$$\Delta U = - W$$

Only ***changes*** in the potential energy of an object are related to work done by forces on the object or to changes in its kinetic energy; hence, the reference point at which U = 0 is ***arbitrary*** and can be conveniently chosen.

Work done by a general variable force (Sec. 7-6):

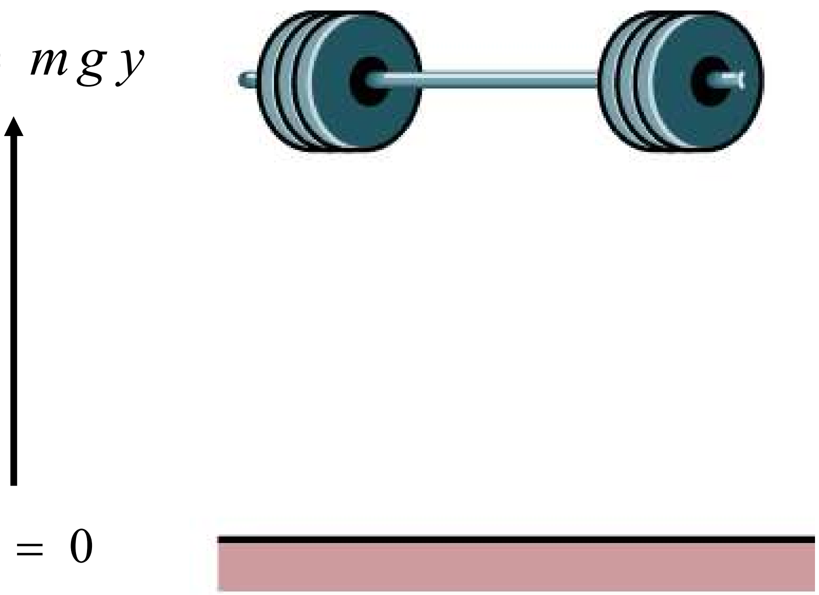
$$W = \int_{x_i}^{x_f} F(x)\,dx$$

Hence: $$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx$$

Gravitational Potential Energy Values

<u>Gravitational potential energy</u> – energy in a set of separated objects which attract one another via the gravitational force.

$$U = mgy$$



$$U = 0$$

the system = earth + barbell

$$\Delta U = -\int_{y_i}^{y_f} F(y)\,dy$$

$$F(y) = -mg$$

$$\Delta U = +mg\int_{y_i}^{y_f} dy = mg\Delta y$$

$$U = mgy$$

Elastic Potential Energy Values

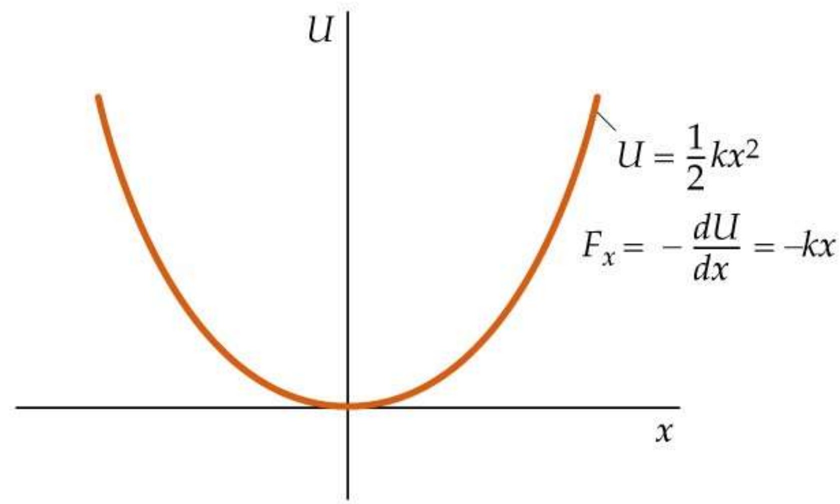
$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx$$

$$F(x) = -k\,x$$

$$\Delta U = +k\int_{x_i}^{x_f} x\,dx = \tfrac{1}{2}k\,x_f^2 - \tfrac{1}{2}k\,x_i^2$$

If U=0 at the relaxed length:

$$U = \tfrac{1}{2}k\,x^2$$



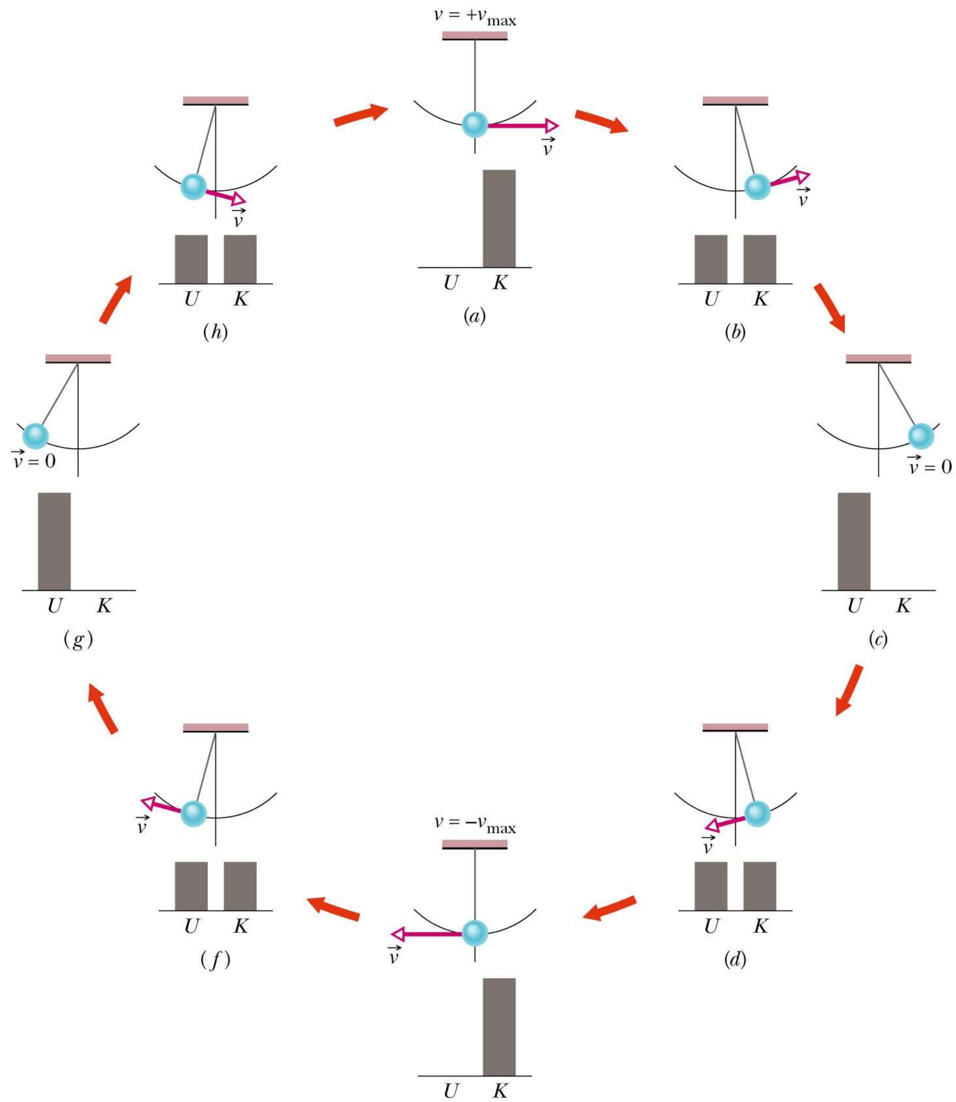
Conservation of Mechanical Energy

Conservation of Mechanical Energy

Work-KE Theorem: $\Delta K = W$

Definition: $\Delta U \equiv -W$

$$\therefore \quad \Delta K = -\Delta U$$
$$K_2 - K_1 = -(U_2 - U_1)$$
$$K_2 + U_2 = K_1 + U_1$$
$$\Delta(K + U) = 0$$



Total Mechanical Energy:

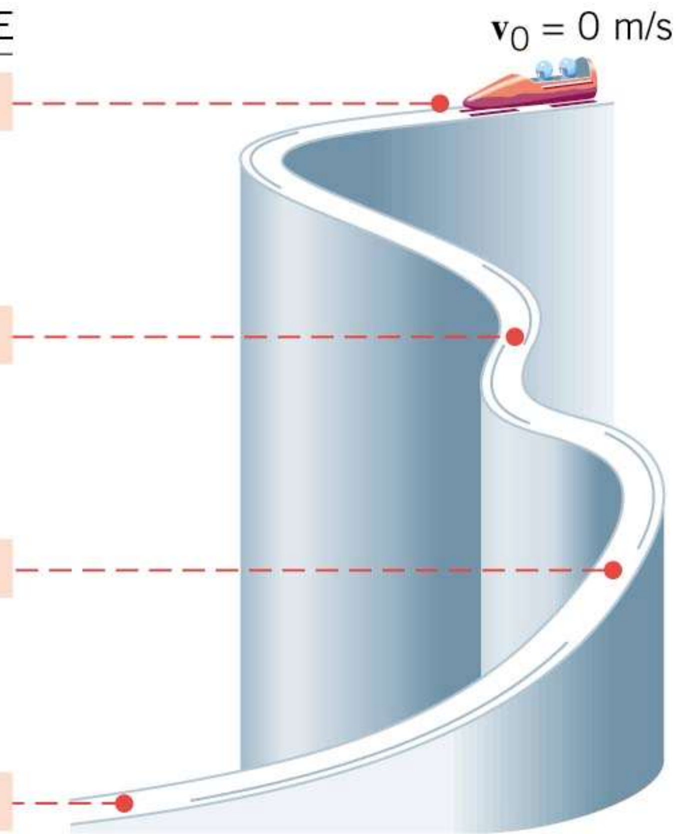
$$E_{mec} = K + U$$

$$\boxed{\Delta E_{mec} = 0}$$

Total Mechanical Energy is conserved.

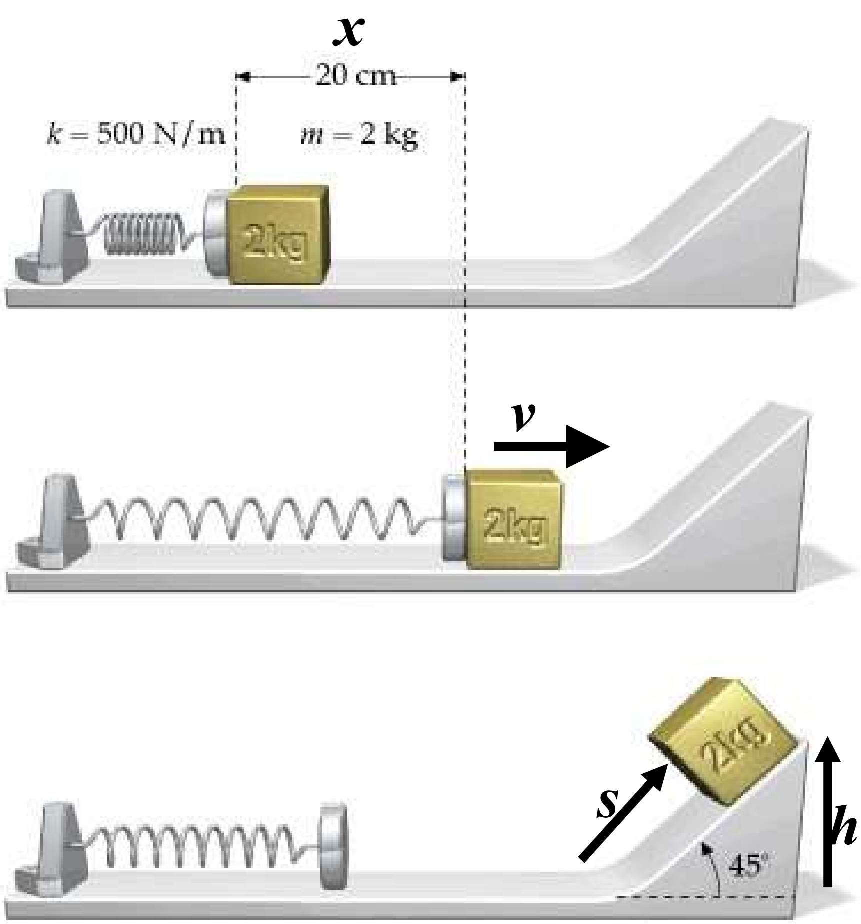
Conservation of Mechanical Energy

KE	PE	E = KE + PE
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J



Cutnell p 168

Example: Energy is conserved

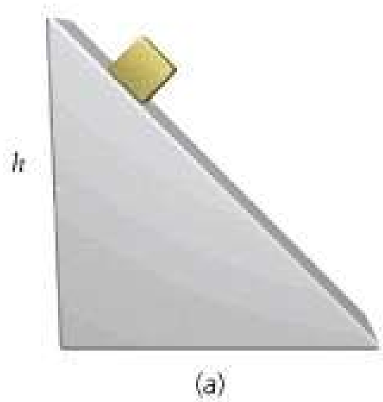


$$E = \tfrac{1}{2} k x^2$$

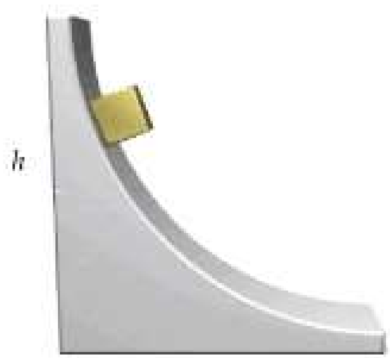
$$E = \tfrac{1}{2} m v^2$$

$$E = m g h = m g s \sin\theta$$

An Alternative to Newton's Laws



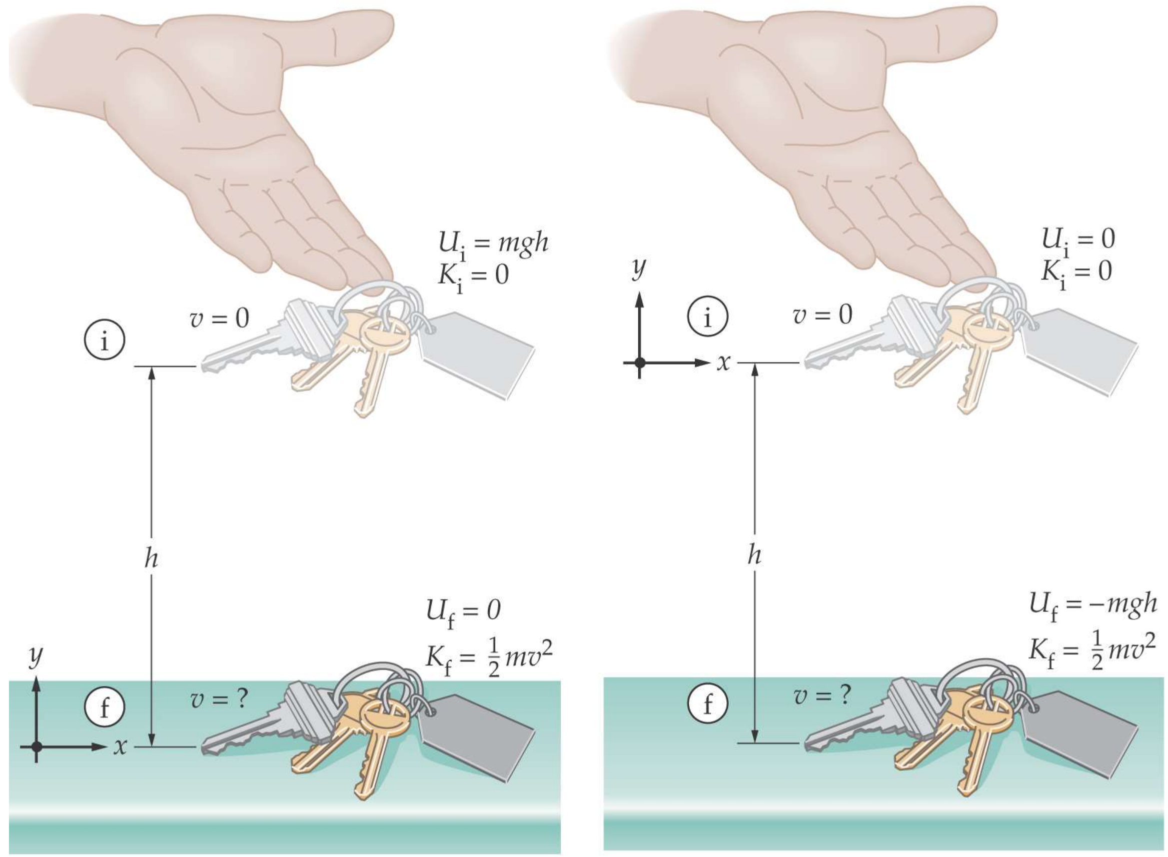
Solve using vector forces and kinematics.



Easier to solve using conservation of energy.

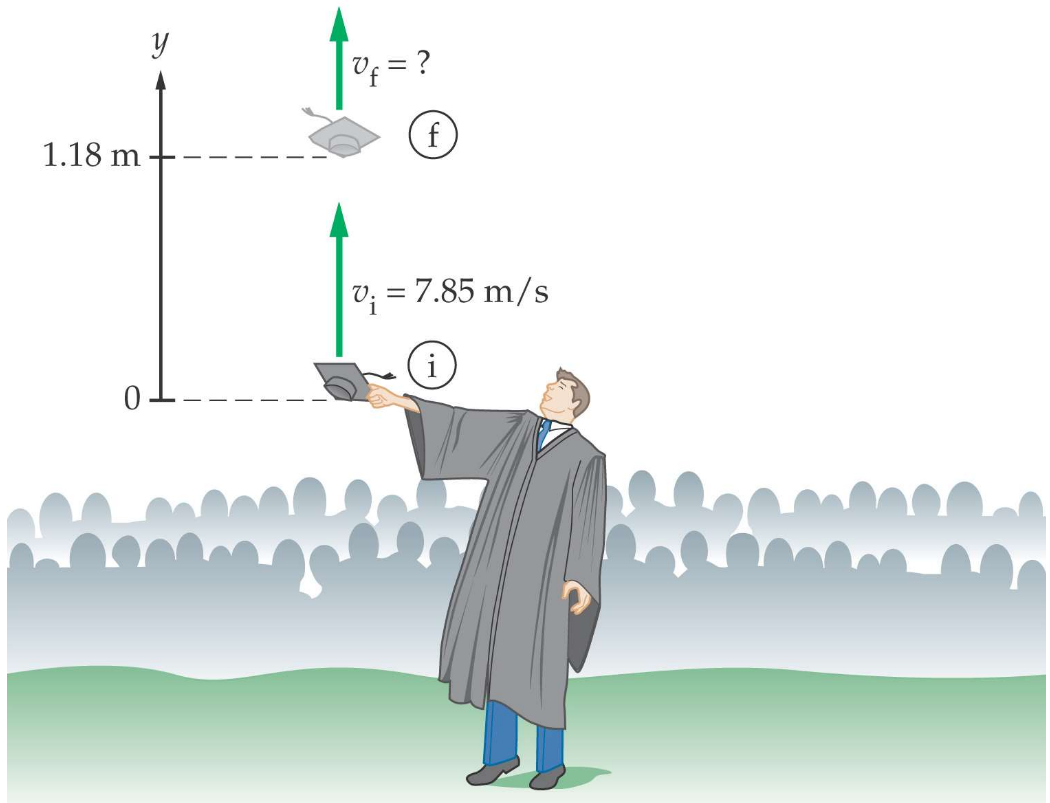
(frictionless surfaces)

Solving a Kinematics Problem Using Conservation of Energy

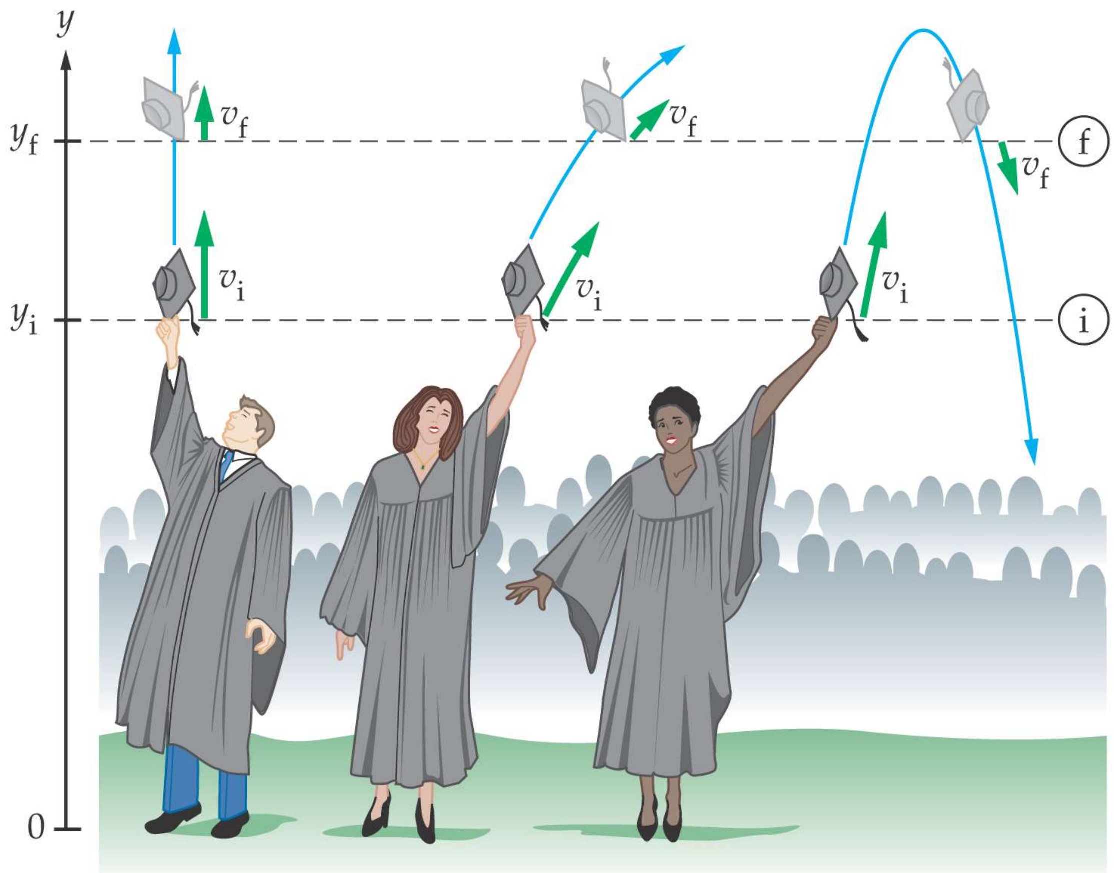


Graduation Fling

m = 0.120 kg
v_i = 7.85 m/s
v at 1.18 m ?

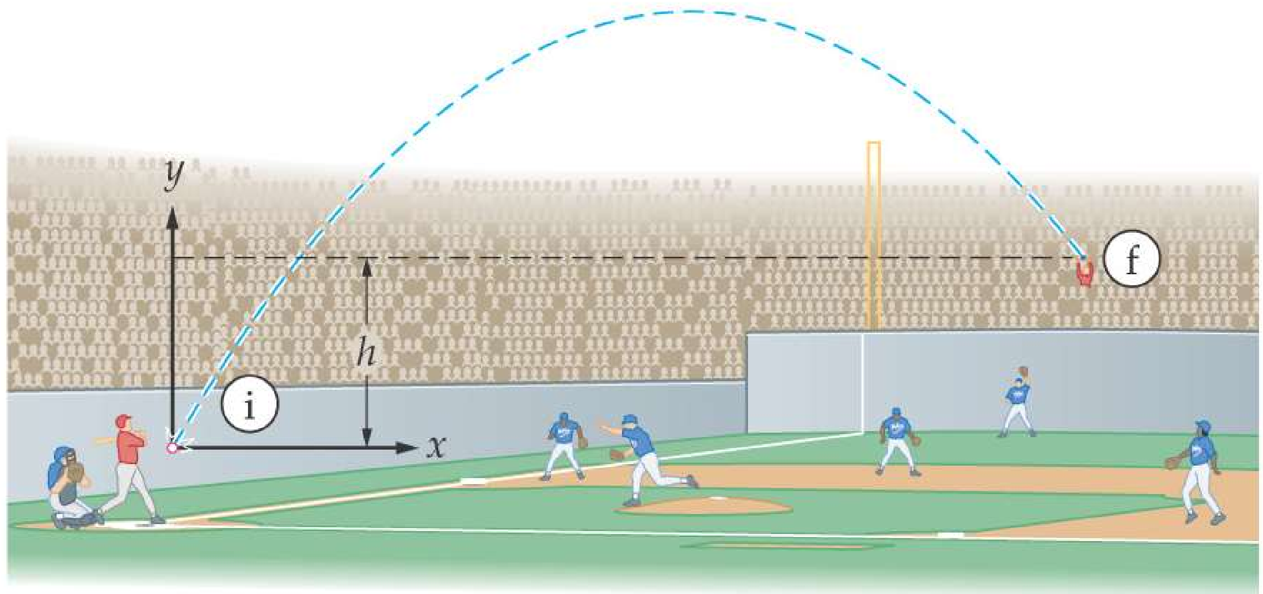


Speed Is Independent of Path

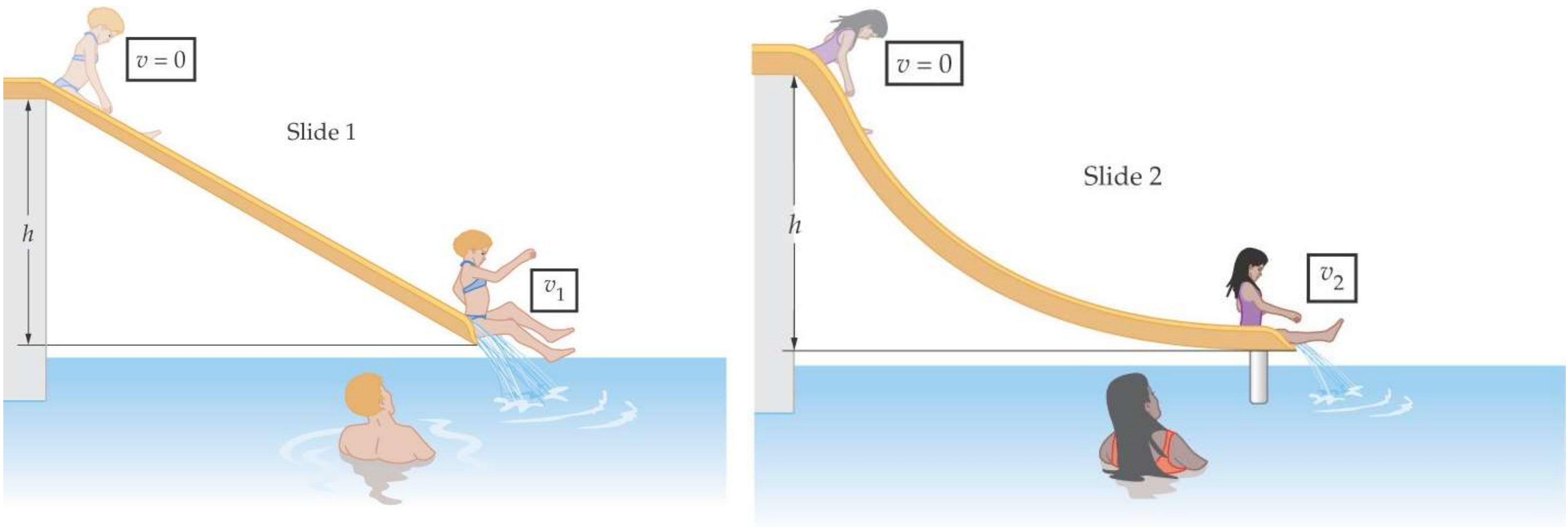


Catching a Home Run

m = .15 kg
v_i = 36 m/s
H = 7.2 m
KE when caught
Speed when caught

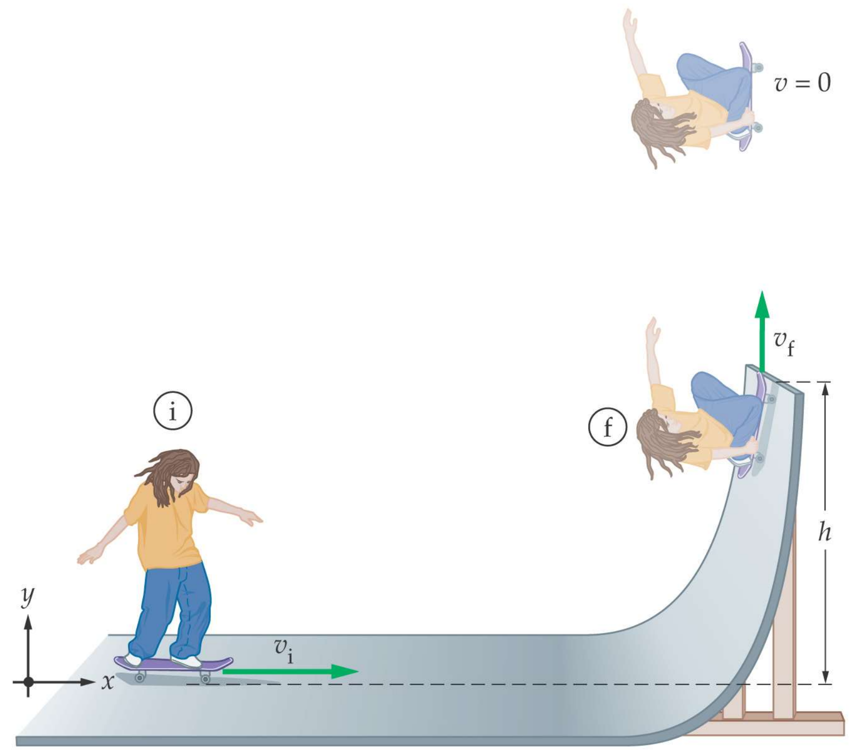


Who is faster at the bottom?



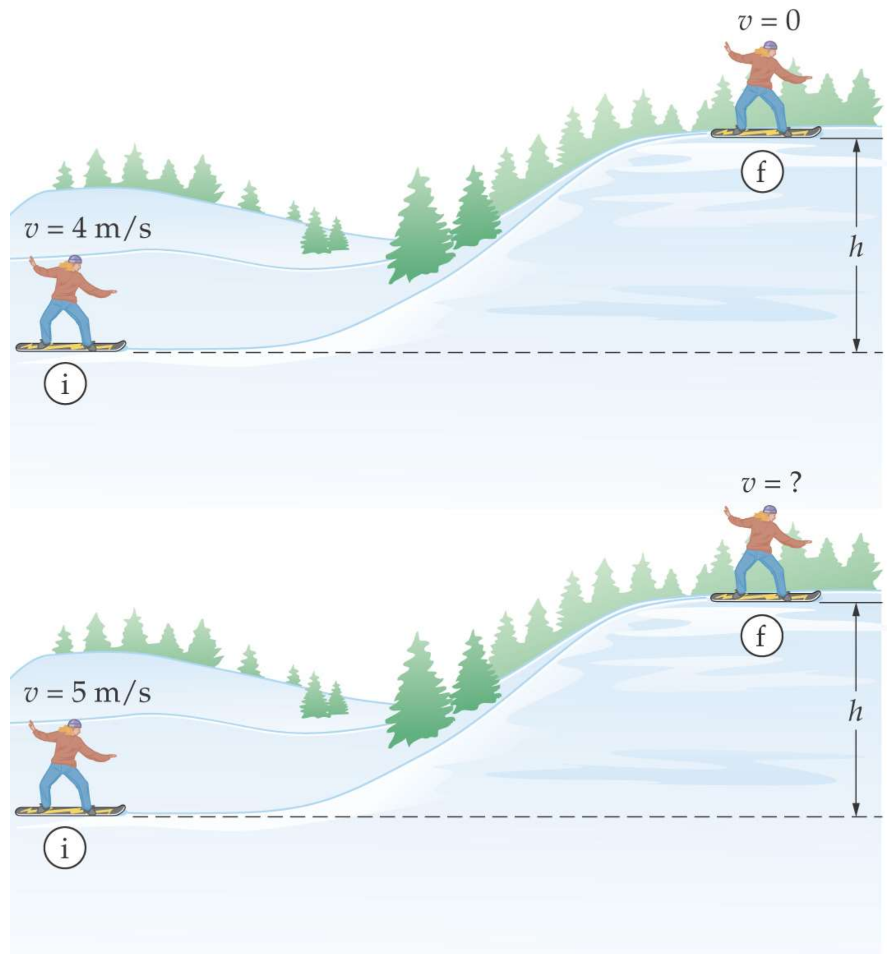
Skateboard Exit Ramp

M = 55 kg
v_i = 6.5 m/s
v_f = 4.1 m/s
h = ?



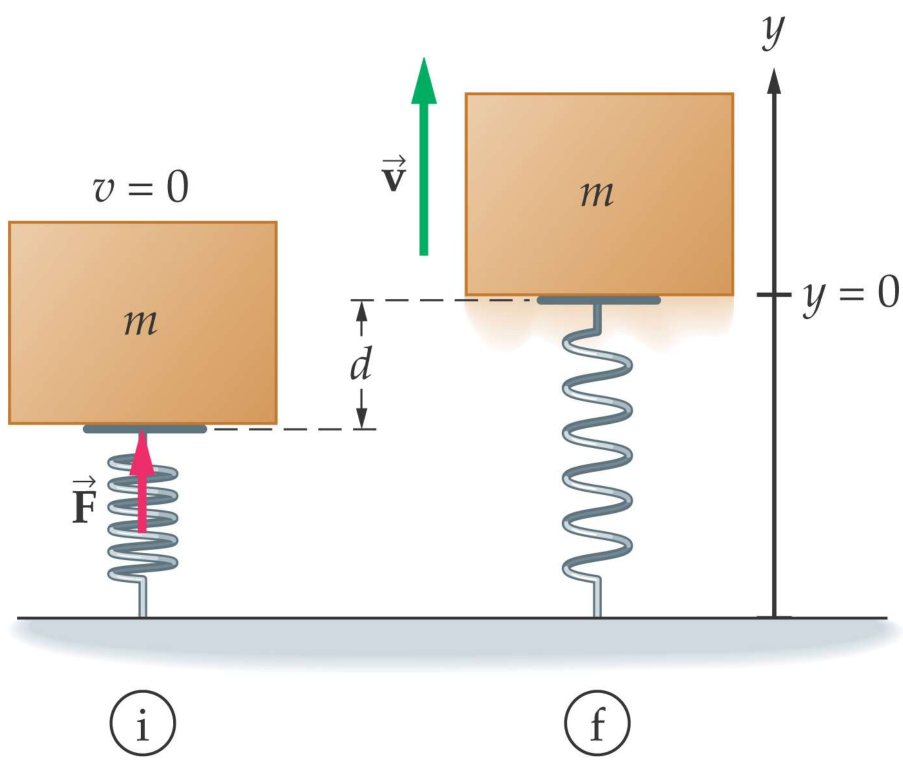
What is the final speed?

Snowboarder starts at 4 m/s, v = 0 at top
Snowboarder starts at 5 m/s, v = ?



Find the Speed of the Block

m = 1.70 kg
k = 955 N/m
Compressed 4.60 cm
v at equilibrium position

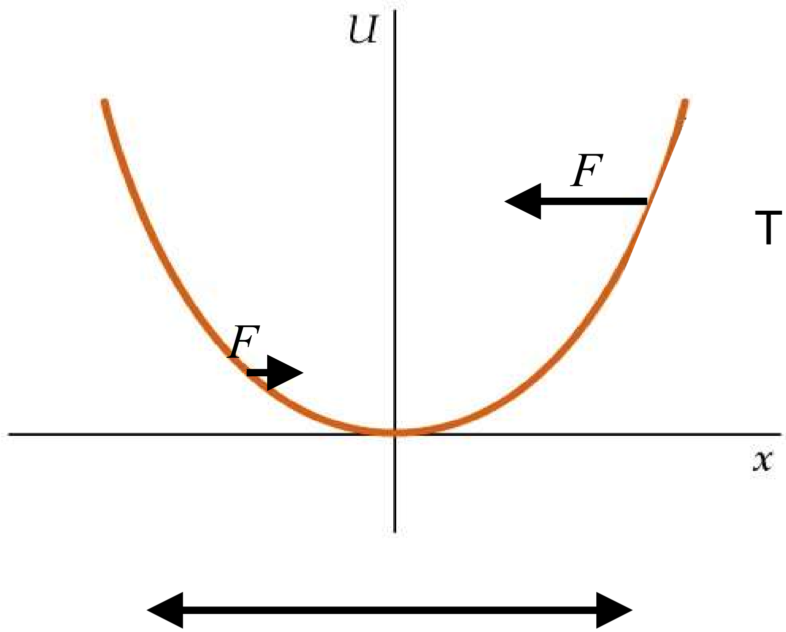


Reading a Potential Energy Curve

Consider the energy of a particle subject to an elastic force:

$$F = -k\,x$$

$$U = \tfrac{1}{2}k\,x^2$$



The particle is trapped.

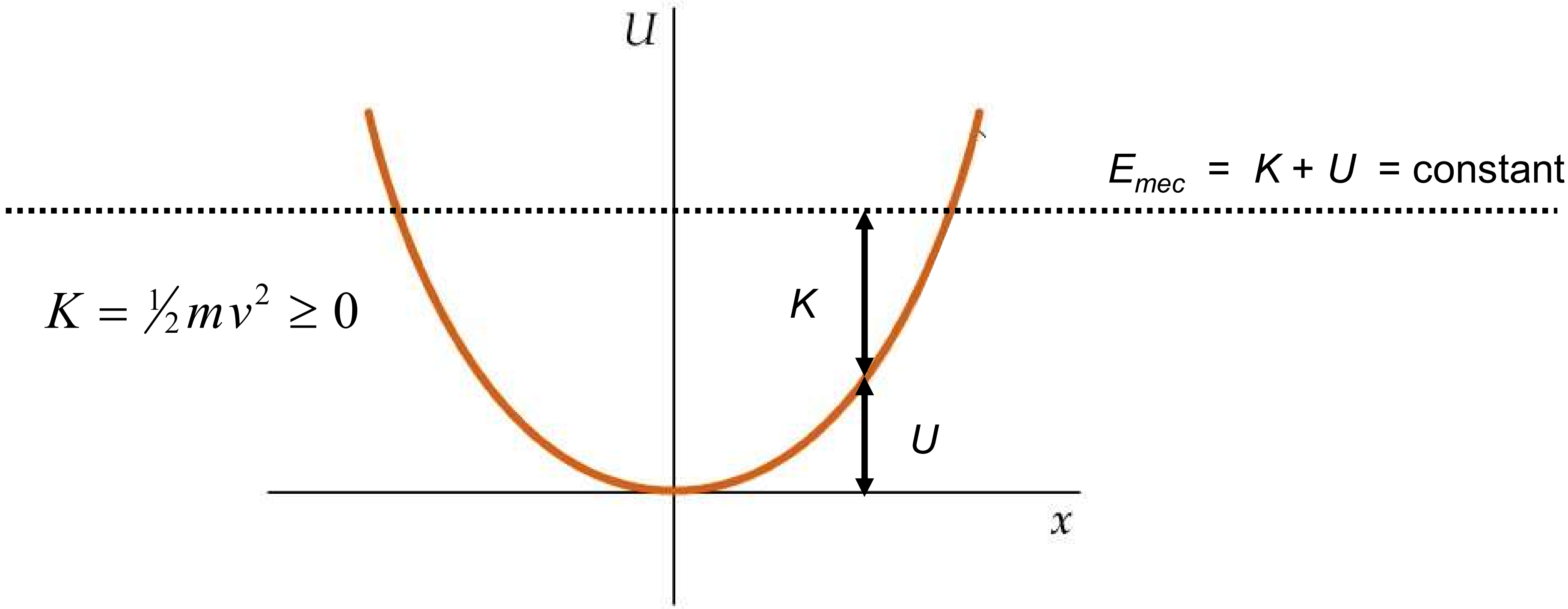
U=0 at the relaxed length.

The range of motion of the particle depends U_{max}.

Reading a Potential Energy Curve

A particle subject to a conservative force; e.g. an elastic force: $F = -k\,x$

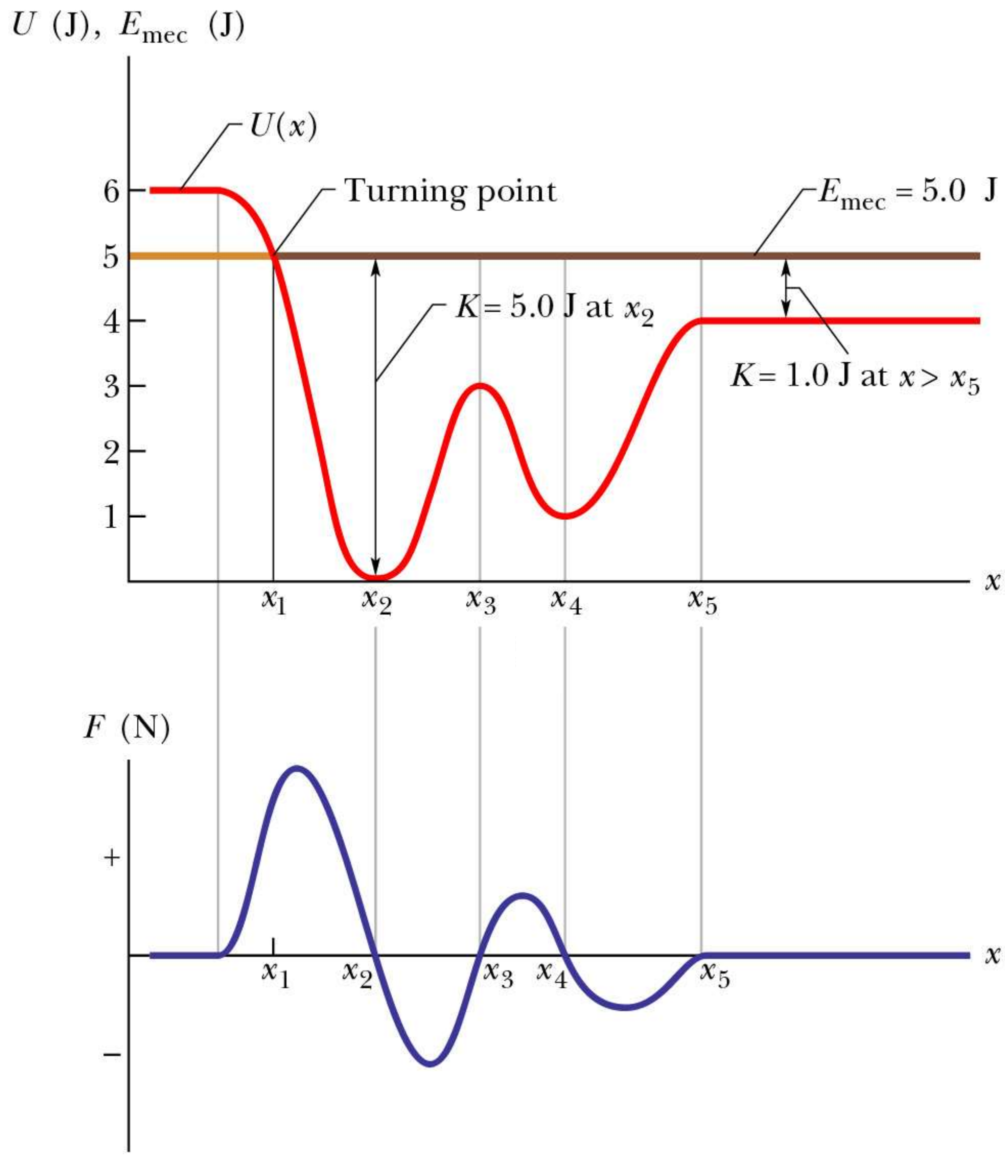
The total mechanical energy of the particle is a constant. $U = \frac{1}{2} k\,x^2$



$K = \frac{1}{2} m v^2 \geq 0$

"*Turning Points*" when $K = 0$

Reading a Potential Energy Curve



$$\Delta U = -W = -F(x)\Delta x$$

$$F(x) = -\frac{dU(x)}{dx}$$

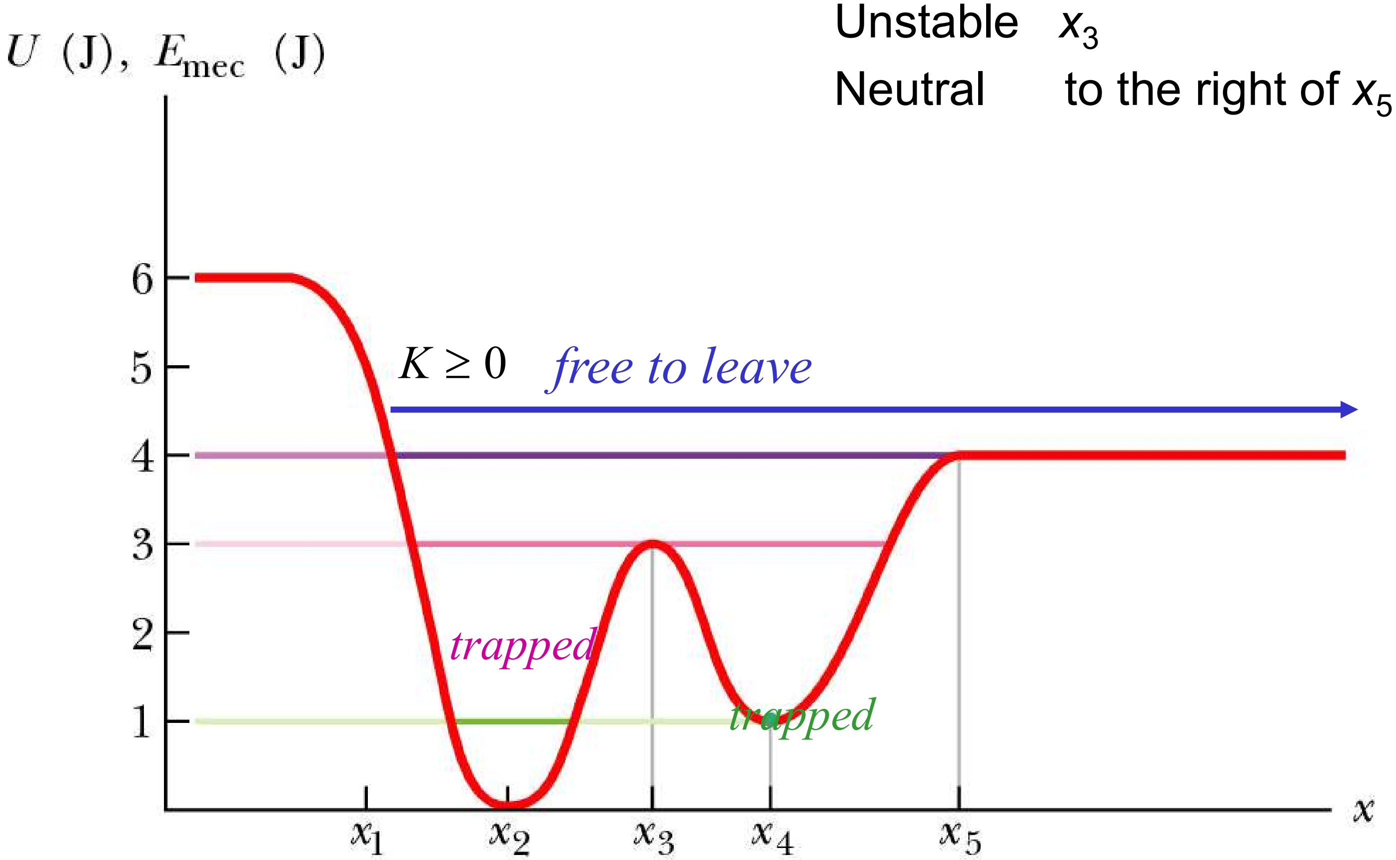
Reading a Potential Energy Curve

Equilibrium: F = 0 and K = 0

Stable x_2 and x_4

Unstable x_3

Neutral to the right of x_5

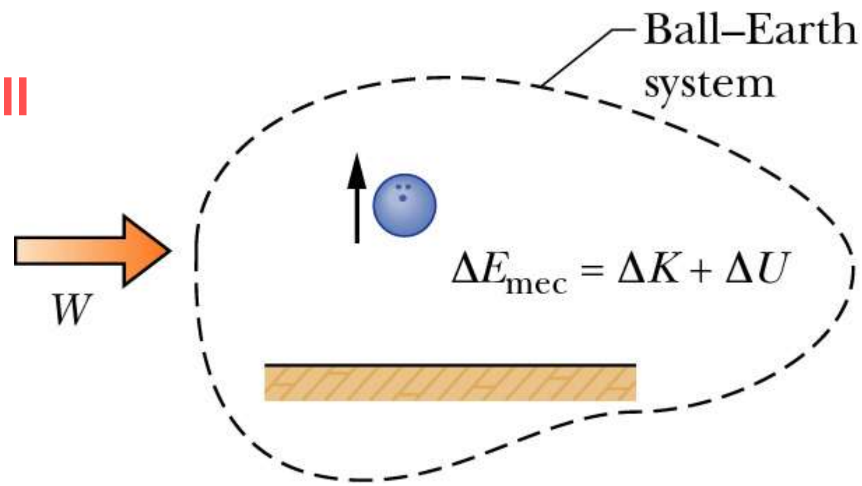


Work Done on a System by an External Force

Work is energy transferred to or from a system by means of an external force acting on that system.

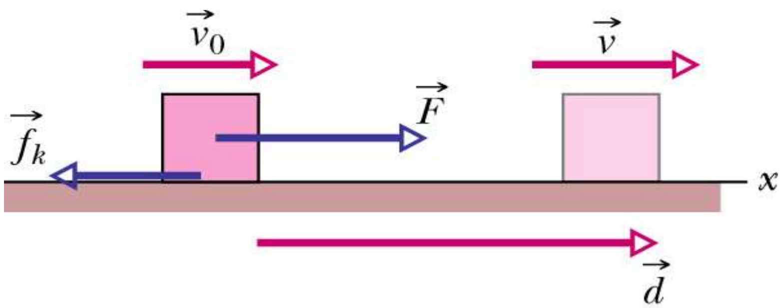
$$W = \Delta E_{mec} = \Delta K + \Delta U$$

Lifting the bowling ball



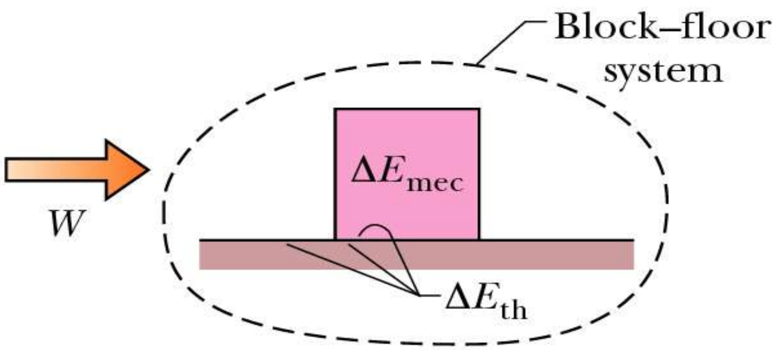
changes the energy of the earth-ball system.

Work Done on a System by an External Force



$$F - f_k = ma$$

$$v^2 = v_0^2 + 2ad$$



$$Fd = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 + f_k d$$

(Thermal Energy)

$$W = \Delta K + \Delta E_{th}$$

Work = change in motion + heat

(e.g. rubbing your hands together)

Generalizing $W = \Delta E_{mec} + \Delta E_{th}$

Non-Conservative Forces

- What are some non- conservative forces?
  - frictional forces
  - air resistance

$$W = W_c + W_{nc} = -\Delta U + W_{nc} = \Delta K$$

$$W_{nc} = DU + DK$$

$$W_{nc} = \Delta E$$

- Summary:

$$W_{total} = \Delta K$$
$$W_c = -\Delta U$$
$$W_{nc} = \Delta E$$

Conservation of Energy

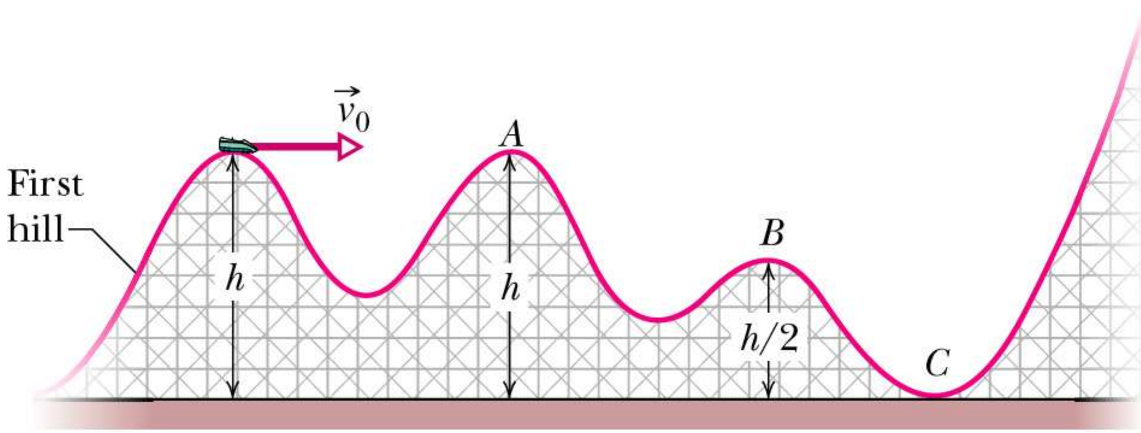
The *total energy of* a system can change only by amounts of energy that are transferred to or from the system.

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

The total energy of an *isolated system* cannot change.

$$\Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$$

In an isolated system, we can relate the total energy at one instant to the total energy at another instant without considering the energies at intermediate times.

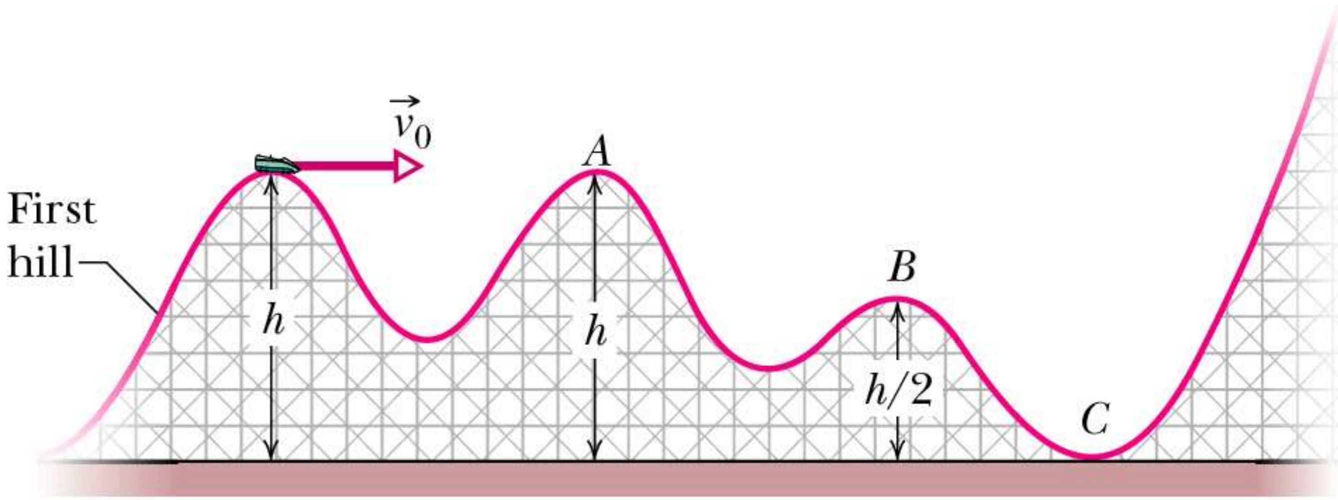


Conservation of Energy

Roller coaster without friction as an isolated system

$$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$$

$$E_{mec} = ½ m v^2 + mgh = \text{constant}$$



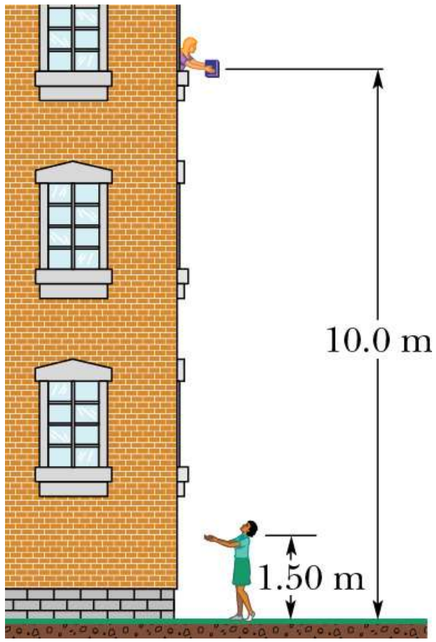
<u>At Point A</u>: $½ m v_0^2 + mgh = ½ m v_A^2 + mgh = \text{constant}$

<u>At Point B</u>: $½ m v_0^2 + mgh = ½ m v_B^2 + mg(h/2)$

<u>At Point C</u>: $½ m v_0^2 + mgh = ½ m v_C^2$

Drop a Textbook

Drop a 2 kg textbook:



a) W_g

b) ΔU_{gp}

c) U_{10}

d) $U_{1.5}$

If $U_0 = 100$ J:

e) W_g

f) ΔU

g) U_{10}

h) $U_{1.5}$

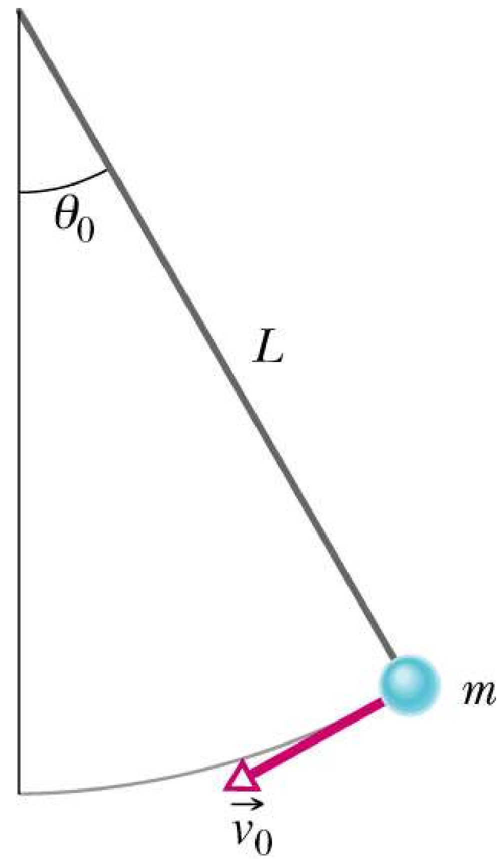
Pendulum Problem

bob has a speed v_0 when the cord makes an angle θ_0 with the vertical.

Find an expression for speed of the bob at its lowest position

Least value of v_0 if the bob is to swing down and then up to

a) Horizontal position
b) Straight up vertically

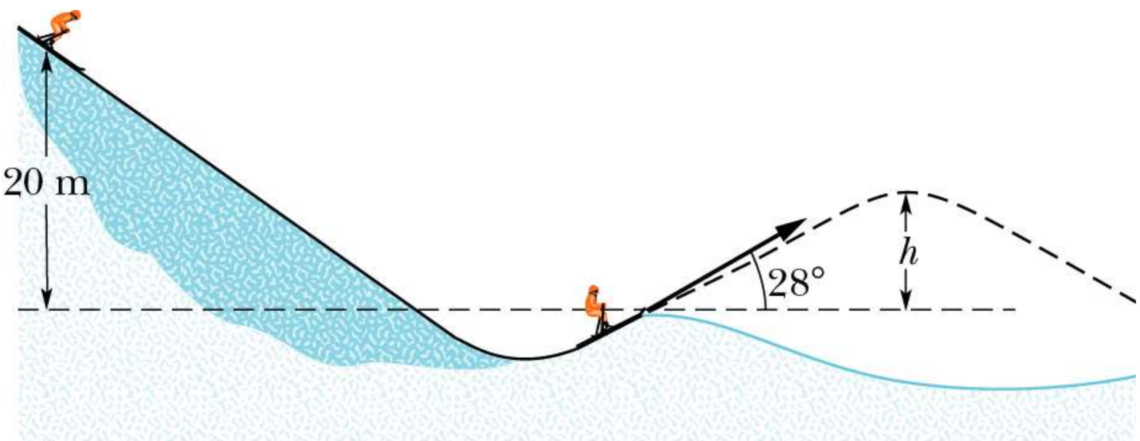


Skier

60 kg skier starts at rest at a height of 20.0 m above end of a ski jump ramp.

As skier leaves ramp, velocity is at an angle of 28^0 with horizontal.

Maximum height?
With backpack of mass 10 kg?



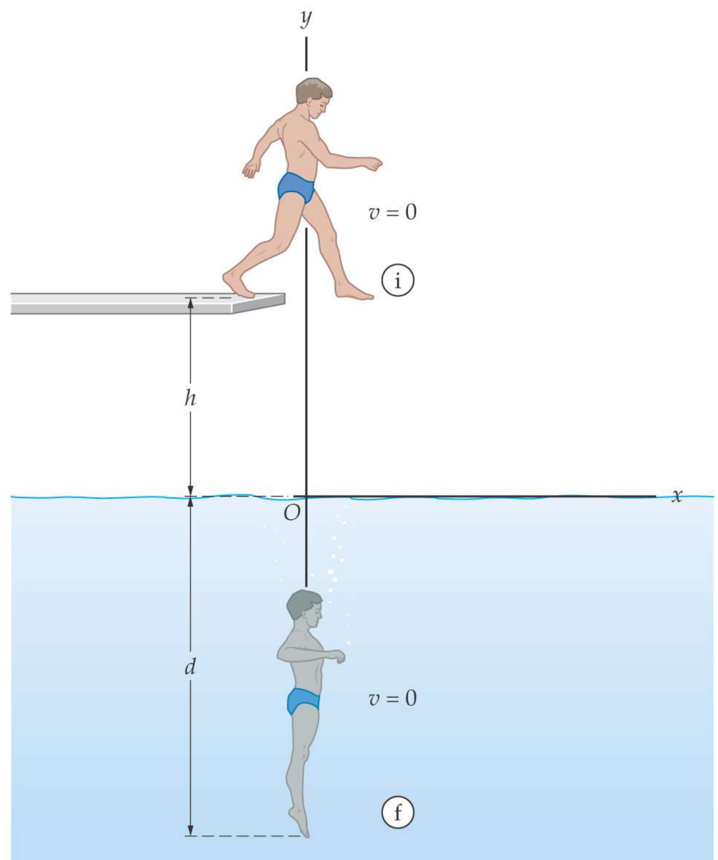
Find the Diver's Depth

$m = 95.0$ kg

$h = 3.0$ m

$W_{nc} = -5120$ J

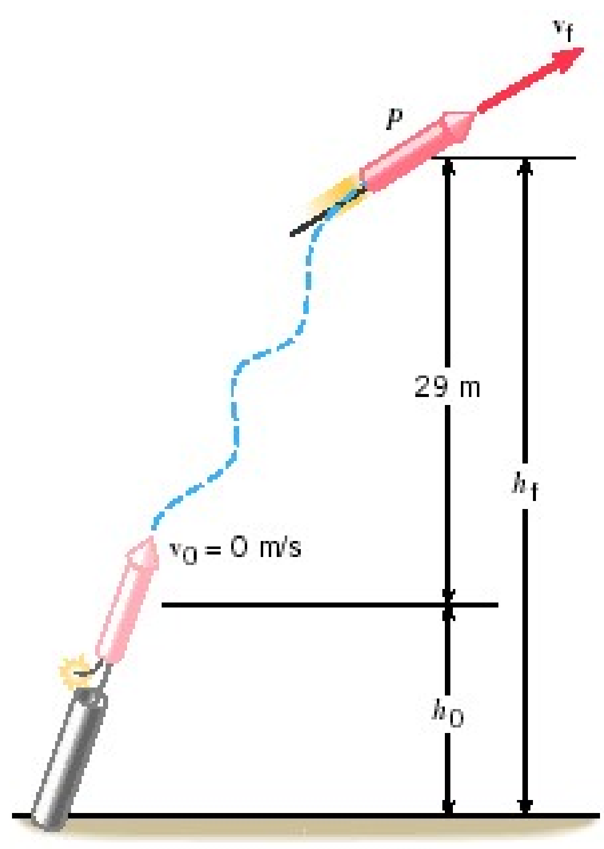
$d = ?$



Nonconservative Forces

$W_{nc} = (½\, mv_f^2 + mgh_f) - (½\, mv_o^2 + mgh_0)$

0.20 kg rocket
$h_f - h_0 = 29$ m
425 J of work is done by propellant
v_f = ?



A Potential Problem

m = 1.60 kg
At x = 0, v = 2.30 m/s
v at x = 2.00 m ?