Chapter 7 Kinetic Energy & Work

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2. Work
3. Work & Kinetic Energy
4. Work Done by a Gravitational Force
5. Work Done by a Spring Force
6. Work Done by a General Variable Force
7. Power

Review & Summary
Questions
Exercises & Problems

Energy & Work

Energy comes in many forms:

- Thermal, Chemical, Atomic, Nuclear Energies

- Energy is ***conserved***.
  - A new tool to solve problems.

- Descriptions of Energy:
  - *Potential Energy* (Chapter 7)
  - ***Kinetic Energy*** - motion of a massive object

$$Kinetic\ Energy \equiv \tfrac{1}{2} m v^2$$

$$\text{Units:} \quad 1\ Joule \equiv 1\ \frac{kg\ m^2}{s^2}$$

August 10, 1972, a large meteorite skipped across the atmosphere.

$$m \approx 4\times10^6\,kg$$
$$v \approx 15\ {}^{m}\!/\!{}_{s}$$

$$\Delta KE = \tfrac{1}{2}mv^2 - 0 = 5\times10^{14}\,J$$

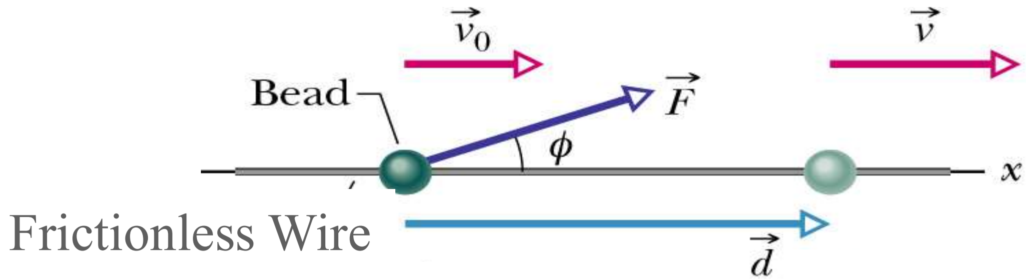
$$\Delta KE \approx 0.1MT \approx 8\ WWII\ atomic\ bombs$$

“Work”

- **<u>Work</u>** (W) is energy transferred to or from an object by means of a force acting on the object.
  - Energy transferred ***to*** the object is ***positive*** work.
    - Stepping on the gas causes positive work to be done on a car.
  - Energy transferred ***from*** the object is ***negative*** work.
    - Stepping on the brakes causes negative work to be done on a moving car.

Work & Kinetic Energy

Consider a force acting on an object constrained to move in the x-direction:



$$v^2 = v_0^2 + 2a_x d$$

$$v^2 - v_0^2 = 2a_x d$$

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = m a_x d$$

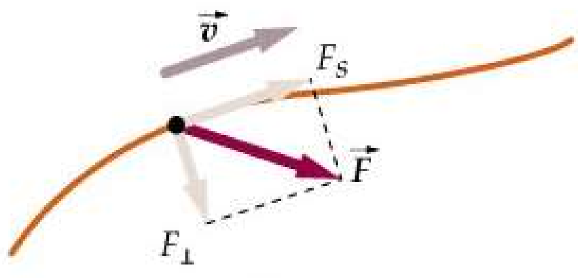
$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = F_x d$$

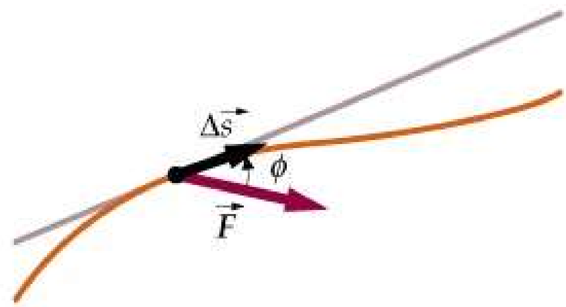
The force causes a change in kinetic energy; *the force does* ***work***:

$$\text{"}Work\text{"} \equiv F_x\, d = F \cos\phi\, d$$

Units: 1 ***Joule*** *= 1 kg (m / s)*^{2} *= (1 kg m / s*^{2}*) m = 1* ***N m***

A force acting on a particle moving along a curve:





$F_\perp$ does no work.

$F_\perp$ does not change the velocity.

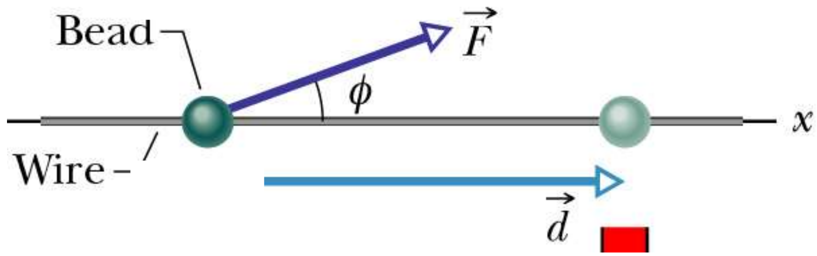
$$\cos\phi = \cos 90^0 = 0$$

$$Work = F_s \, \Delta s$$

$$Work = F \cos\phi \, \Delta s$$

$$Work = \vec{F} \bullet \overrightarrow{\Delta s}$$

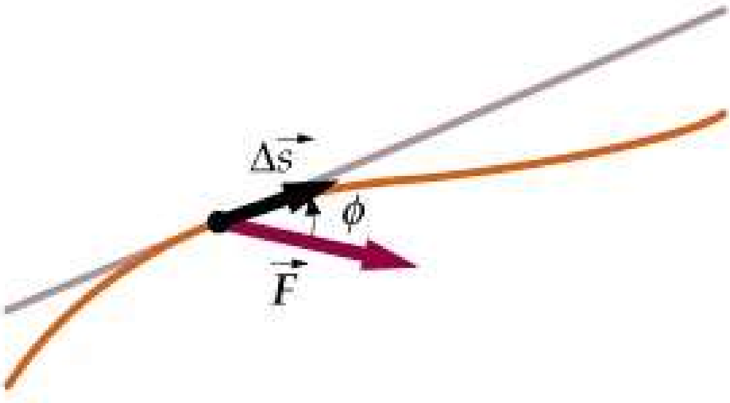
"Work"



$$W \equiv \vec{F} \bullet \vec{d} = F \cos\phi \; d$$

- **Work** (W) is energy transferred to or from an object by means of a force acting on the object.
  - A force does ***positive*** work when it has a vector component in the same direction as the displacement.
  - A force does ***negative*** work when it has a vector component in the opposite direction as the displacement.
  - A force does ***zero*** work when it is perpendicular to the displacement.

Work – Kinetic Energy Theorem



$$\Delta W = F_S \, ds$$

$$W = \int_{s_1}^{s_2} F_S \, ds = \int_{s_1}^{s_2} m \, a_S \, ds$$

$$a_S = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$$

chain rule for derivatives

$$W = \int_{s_1}^{s_2} m\, v \frac{dv}{ds} ds = m \int_{v_0}^{v} v \, dv$$

$$W = \tfrac{1}{2} m v^2 - \tfrac{1}{2} m {v_0}^2$$

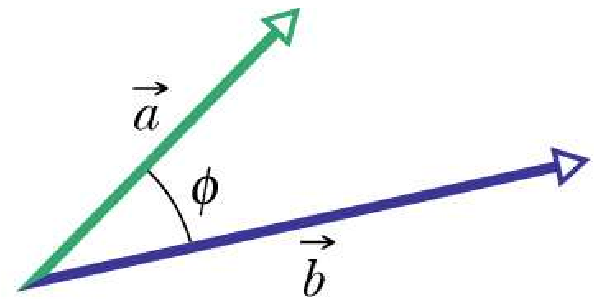
$$W = K_f - K_i$$

$$W = \Delta K$$

Net Work Done = Change in Kinetic Energy

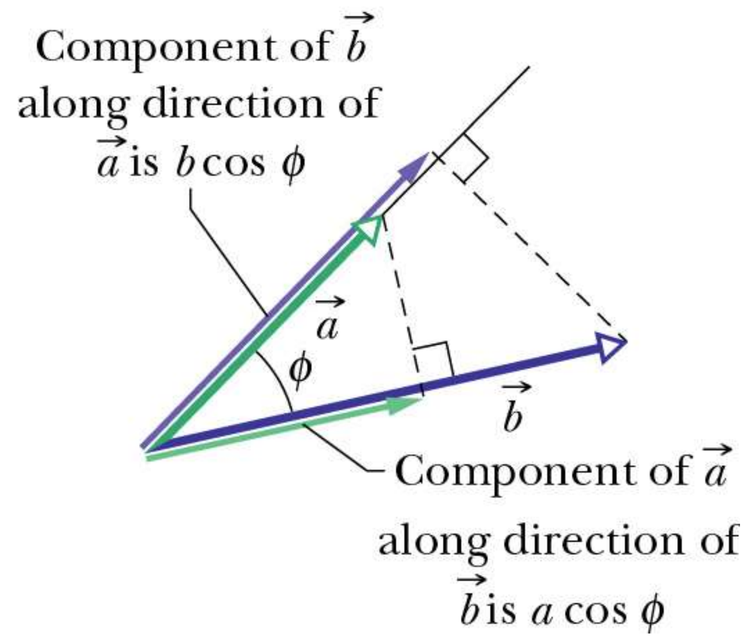
Scalar (Dot) Product of Two Vectors

$$\vec{a} \bullet \vec{b} = a\ b \cos\phi$$



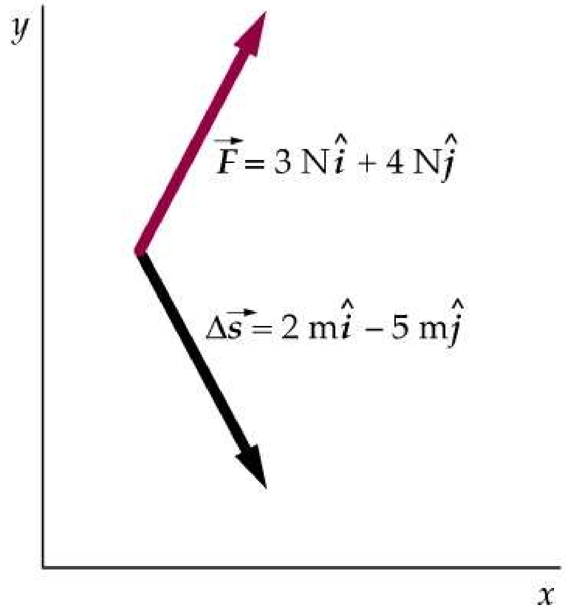
Commutative Law:

$$\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a}$$



$$\vec{a} \bullet \vec{b} = a_x\ b_x + a_y\ b_y + a_z\ b_z$$

Example



$$Work = \vec{F} \bullet \overrightarrow{\Delta s}$$

$$W = F_x\,\Delta x + F_y\,\Delta y$$

$$W = (3N)(2m) + (4N)(-5m)$$

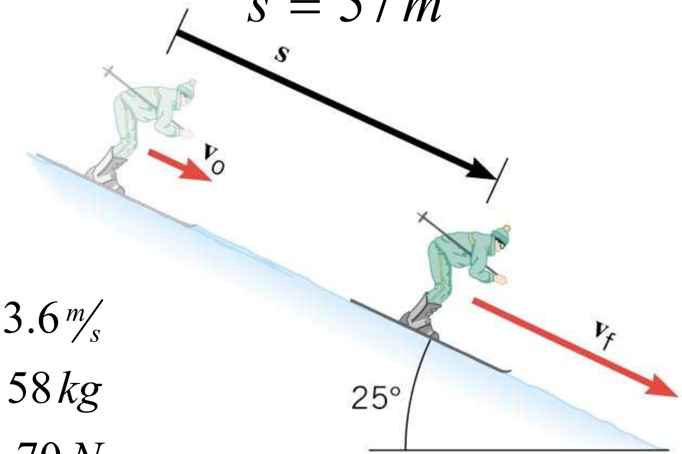
$$W = -14\,Nm$$

Tipler p 157

Example:

v_f = ?

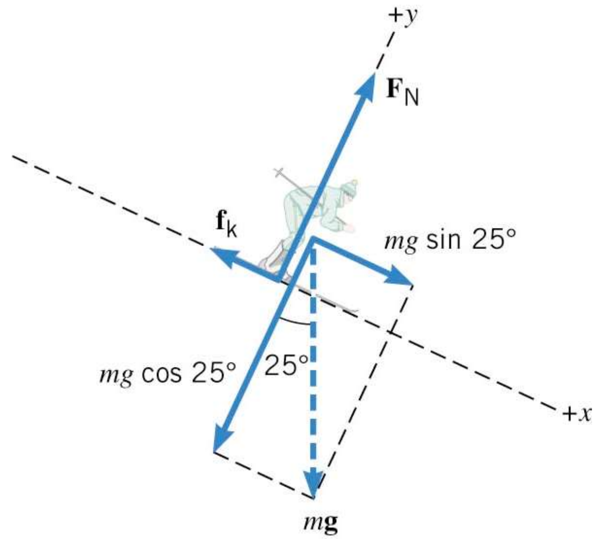
$s = 57\,m$



$v_0 = 3.6\,{}^{m}\!/\!_{s}$

$m = 58\,kg$

$f_K = 70\,N$



$$Work = \vec{F}_{net}\; s$$

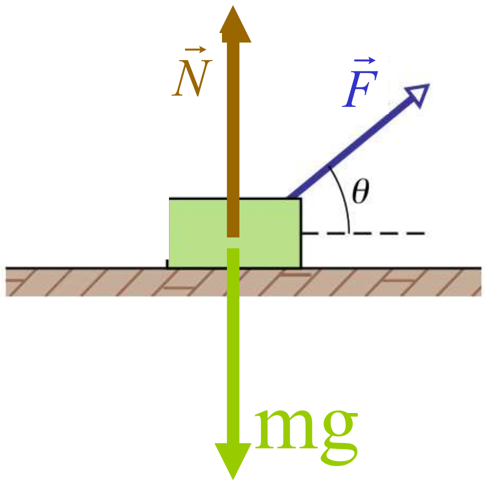
$$\vec{F}_{net} = m\,g\sin 25^0 - f_K$$

$$\left(m\,g\sin 25^0 - f_K\right)s = \tfrac{1}{2}m\,v_f^{\,2} - \tfrac{1}{2}m\,v_0^{\,2}$$

$$v_f = 19\,{}^{m}\!/\!_{s}$$

Cutnell p 160

Work done by the forces on the box?



$$F = 210\ N$$

$$\theta = 20^0$$

$$d = 3\ m$$

$$W_F = \vec{F} \bullet \vec{d}$$

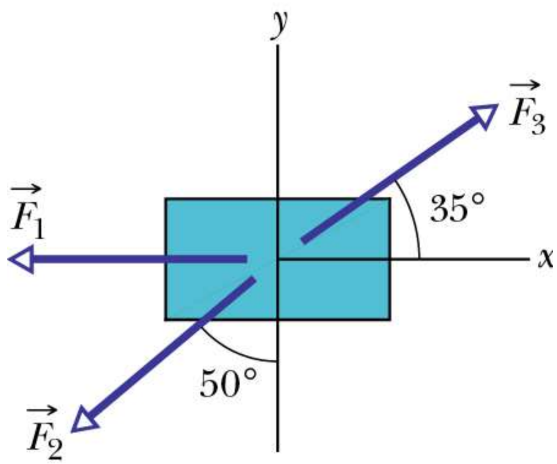
$$W_F = F\,d\,\cos\theta = (210\ N)(3\ m)\cos(20^0) = 600\ J$$

$$W_N = \vec{N} \bullet \vec{d}$$

$$W_N = N\,d\,\cos(90^0) = 0 = W_g$$

Object sliding on a frictionless floor

Work = ?



$$F_1 = 3.00N$$
$$F_2 = 4.00N$$
$$F_3 = 10.0N$$

$$F_{net_x} = -F_1 - F_2 \sin 50^0 + F_3 \cos 35^0$$

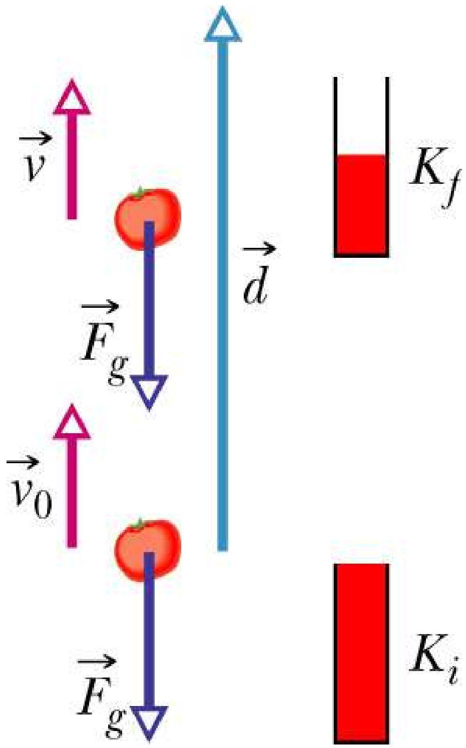
$$F_{net_y} = -F_2 \cos 50^0 + F_3 \sin 35^0$$

$$F_{net}{}^2 = F_{net_x}{}^2 + F_{net_Y}{}^2$$

$$Since\ v_0 = 0,\ d\ is\ parallel\ to\ F_{net}: \quad W = F_{net}\, d$$

$$W = 15.3\ J$$

Work Done ***by the Gravitational Force***



Going down:

$$W = \vec{F} \bullet \vec{d}$$
$$W = m\ g\ d\ \cos 0^0$$
$$W = +m\ g\ d$$

The work done by the gravity force increased the kinetic energy of the object.

Going up:

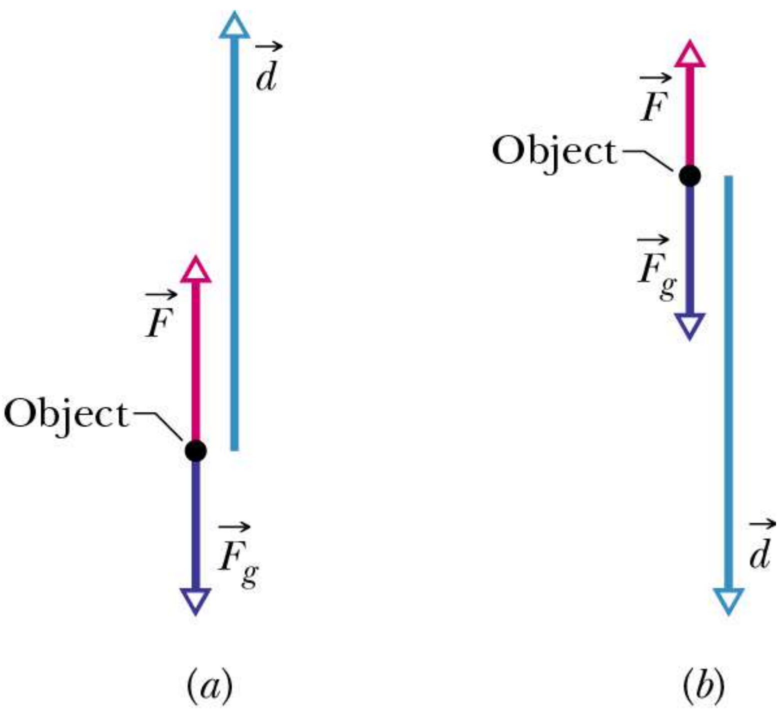
$$W = \vec{F} \bullet \vec{d}$$
$$W = m\ g\ d\ \cos 180^0$$
$$W = -m\ g\ d$$

The work done by the gravity force reduced the kinetic energy of the object.

Work Done Lifting & Lowering an Object

To raise an object to a new location, an *applied force*, ***F***, must act to overcome the *gravity force*, $\boldsymbol{F}_g$.

Consider a case where an object is ***at rest before and after*** the move.



Raising Lowering

$$W_a = mgd \qquad W_a = -mgd$$

Work- Energy Theorem:

$$K_f - K_i = W_a + W_g$$

But $K_f = K_i = 0$:

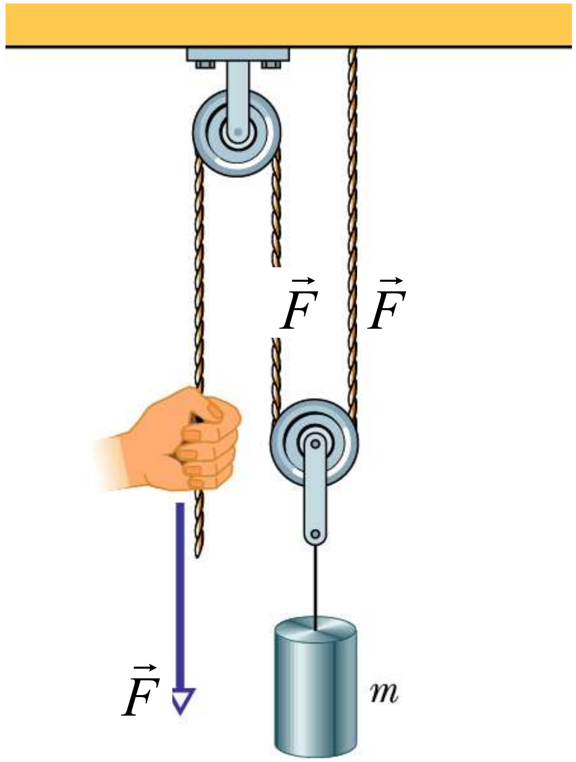
$$0 = W_a + W_g$$

$$W_a = -W_g$$

The work done by the applied force is the negative of the work done by the gravitational force.

The work done by the lifter.

Pull with constant force F.



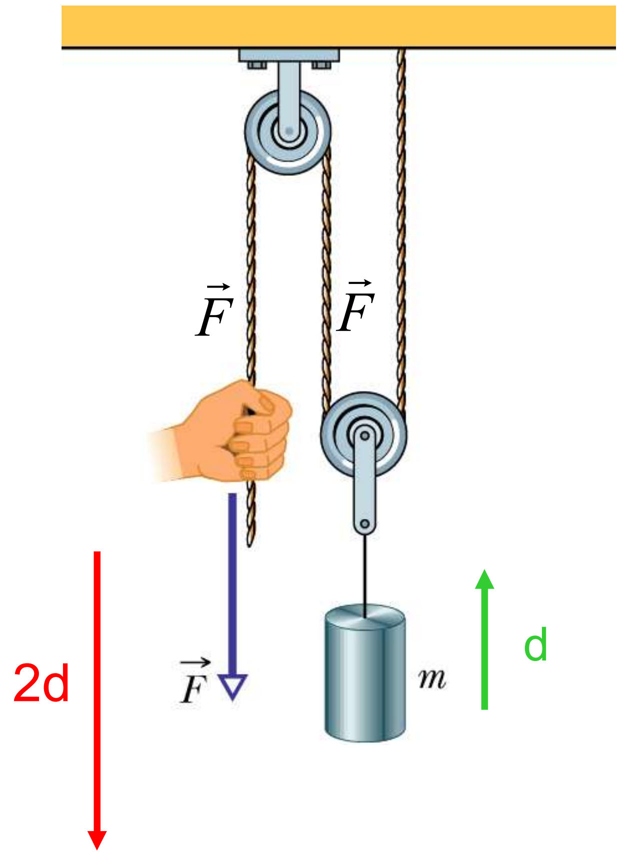
$$\vec{F} = m\vec{a}$$

$$2F - mg = ma = 0$$

$$F = \tfrac{1}{2}mg$$

Is less energy expended or less work done when using a pulley (or other such device)?

Pull with constant force F.



$$\vec{F} = m\vec{a}$$
$$2F - mg = ma = 0$$
$$F = \tfrac{1}{2} mg$$

$$W = (F)(2d)$$
$$W = mgd$$

$$W = (2F)(d)$$
$$W = mgd$$

Same amount of work!

An object accelerated downwards

a = g/4
drops d
Work by cord ?
Work by gravity?
Final velocity

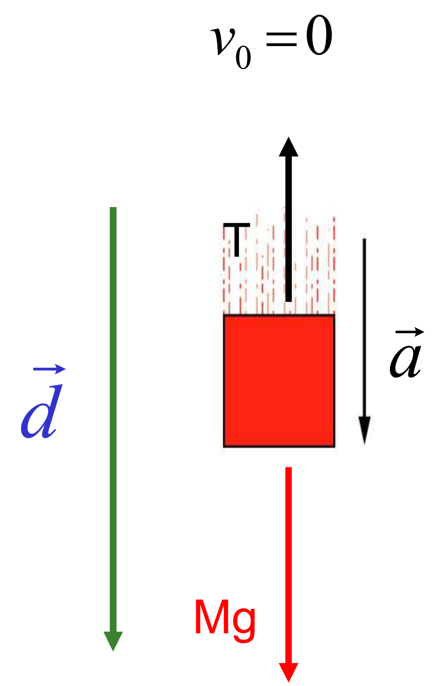
$$\vec{F} = m\vec{a}$$

$$T - mg = M\,a = M\left(-\frac{g}{4}\right)$$

$$T = \tfrac{3}{4}M\,g$$

$$W_T = -\left(\tfrac{3}{4}M\,g\right)d$$

$$W_g = +M\,g\,d \qquad W_{net} = +\tfrac{1}{4}M\,g\,d$$



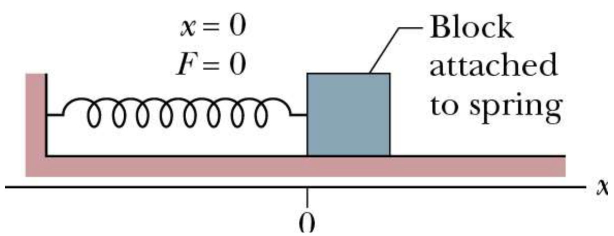
Work Energy Theorem:

$$K - K_0 = W_{net}$$

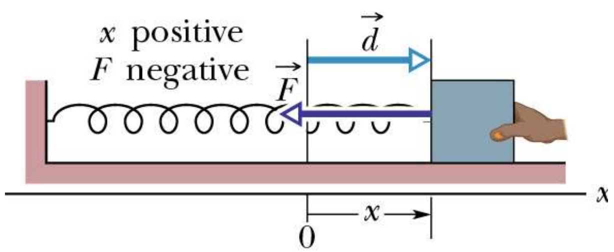
$$\tfrac{1}{2}M\,v^2 - 0 = \tfrac{1}{4}M\,g\,d$$

$$v = +\sqrt{\tfrac{1}{2}g\,d}$$

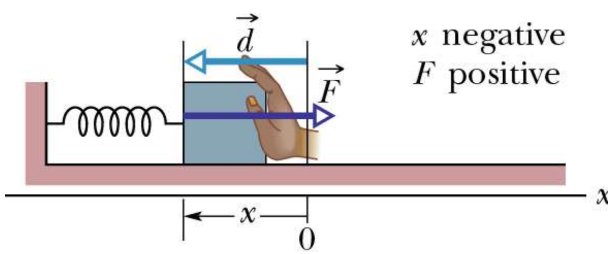
Work Done by a Spring Force



Relaxed



Stretching



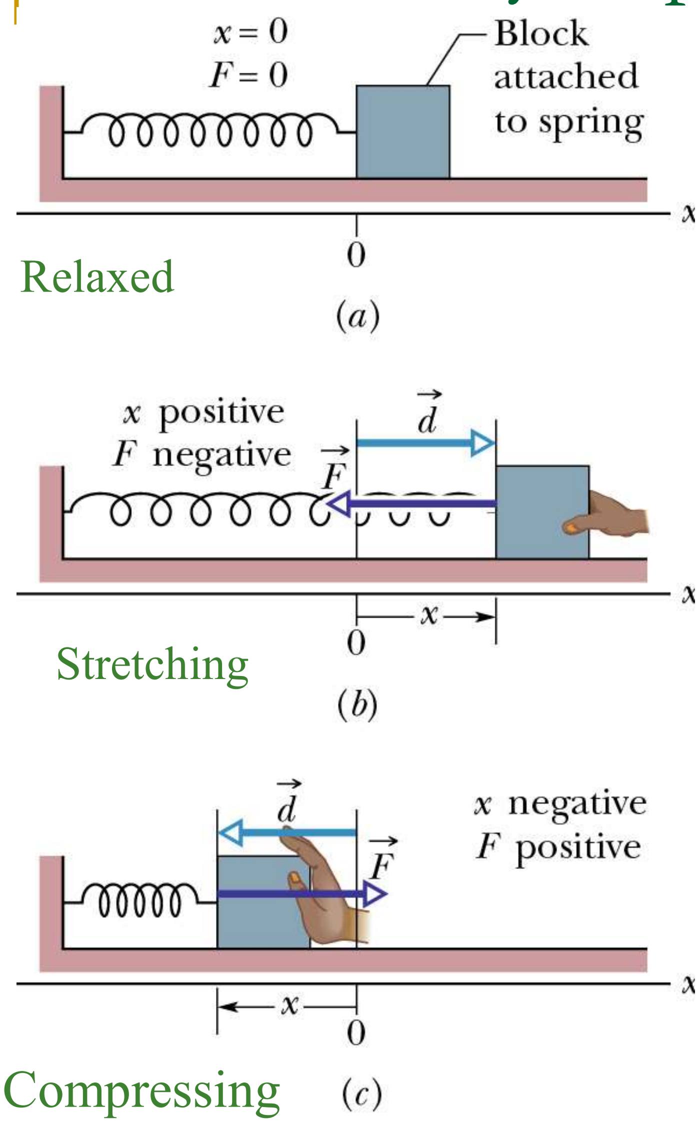
Compressing

<u>Hooke's Law:</u> $\vec{F} = -k\,\vec{d}$

where k = spring constant (N/m)

The force <u>varies</u> linearly with displacement.

Work Done by a Spring Force



Hooke's Law: $\vec{F} = -k\,\vec{d}$

Work done by the spring force:

$$W_S = \int_{x_i}^{x_f} F(x)\,dx$$

$$W_S = -k\int_{x_i}^{x_f} x\,dx = -\tfrac{1}{2}k\,x^2\Big|_{x_i}^{x_f}$$

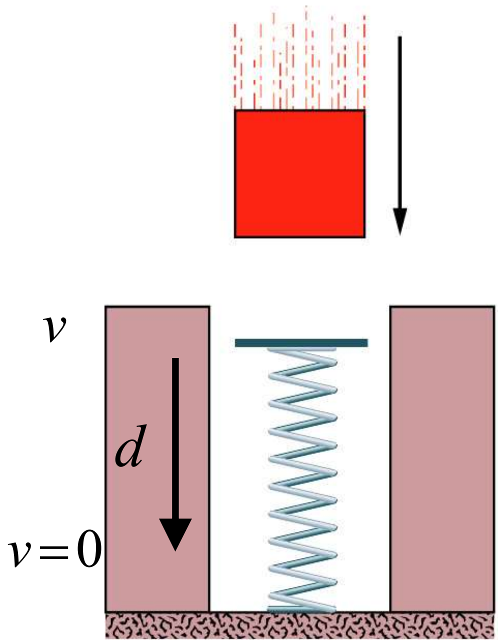
$$W_S = \tfrac{1}{2}k\,x_i^2 - \tfrac{1}{2}k\,x_f^2$$

$$W_S = -\tfrac{1}{2}k\,x^2 \qquad (x_i = 0)$$

$$W_a = -W_S$$

A block is dropped on a relaxed spring

m = 250 g
k = 2.5 N/cm
d = 12 cm
W_g ? W_s?
v ?



$$W_g = m g d$$

$$W_S = -\tfrac{1}{2} k d^2$$

$$W_{net} = W_g + W_S$$

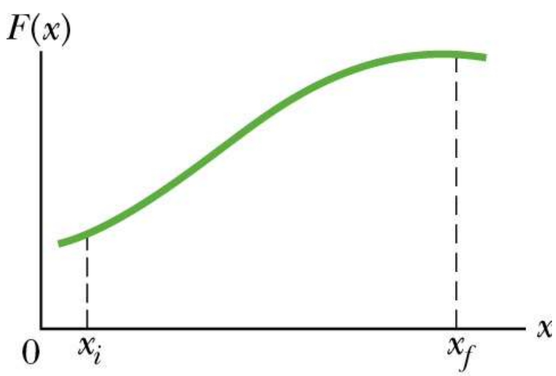
$$W_{net} = K_f - K_i$$

$$m g d - \tfrac{1}{2} k d^2 = 0 - \tfrac{1}{2} m v^2$$

$$v = \sqrt{\left(\frac{k}{m}\right) d^2 - 2 g d}$$

$$v = 3.5 \, m/s$$

Work – Kinetic Energy Theorem with a Variable Force

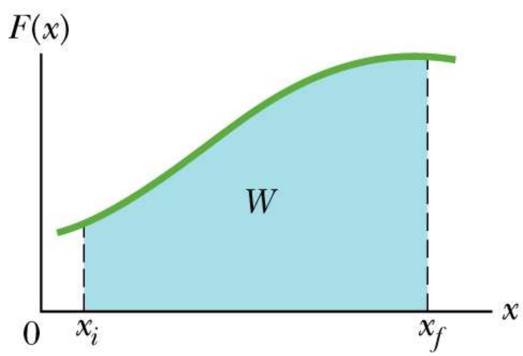


$$W_S = \int_{x_i}^{x_f} F(x)\,dx = m\int_{x_i}^{x_f} a_x\,dx$$

$$a_x\,dx = \frac{dv}{dt}\,dx = \frac{dx}{dt}\,dv = v\,dv$$

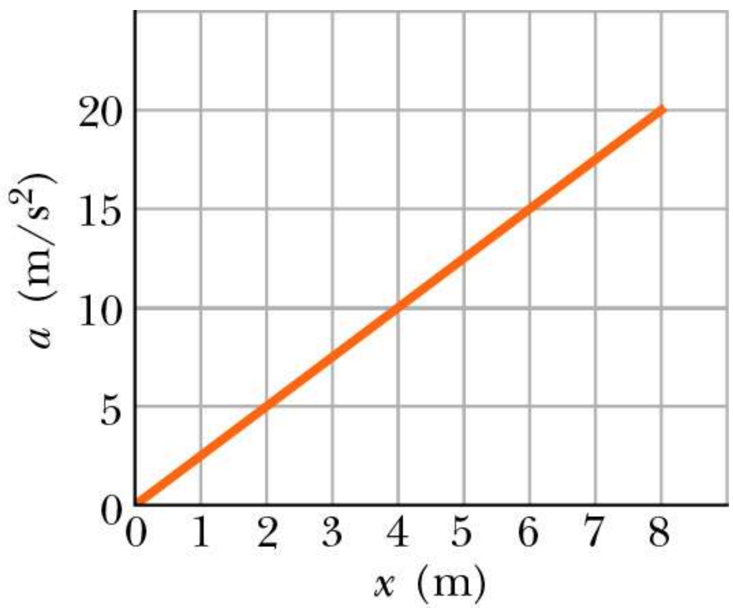
$$W = m\int_{x_i}^{x_f} v\,dv = \tfrac{1}{2} m v^2 \Big|_{v_i}^{v_f}$$

$$W = KE_f - KE_i$$



Problem

A 10 kg brick moves with the following variable acceleration; how much work is done on the brick in the first 8 seconds.



$$a = \alpha x$$

$$\alpha = \frac{20\,{}^{m}\!/\!_{s^2}}{8\,m} = 2.5\,s^{-2}$$

$$a = 2.5\,x$$

$$W_S = \int_{x_i}^{x_f} F(x)\,dx = 2.5m \int_{x_i}^{x_f} x\,dx$$

$$W = {}^{2.5}\!/\!_{2}\, m\,x^2 \Big|_0^8 = {}^{2.5}\!/\!_{2}\,(10)(8)^2 = 800\,J$$

Power

Power = time rate at which Work is done due to a Force.

$$P \equiv \frac{dW}{dt}$$

SI Units: $1\ Watt = 1\ Joule / s = 1\ N\,m / s$

English Units: *horsepower*

$$1\ hp = 550\ ft\ lbs / s = 746\ Watts$$

For a constant force: $P = \frac{d\left(\vec{F} \bullet \vec{d}\right)}{dt} = \vec{F} \bullet \vec{v}$

Work = Power x Time

1 kW hr = 3.6 x 10^6 J