Physics 1

Chapter 4 Motion in 2 &3 Dimensions

1. Moving in 2 &3 Dimensions
2. Position & Displacement
3. Average & Instantaneous Velocity
4. Average & Instantaneous Acceleration
5. Projectile Motion
6. Projectile Motion Analyzed
7. Uniform Circular Motion
8. Relative Motion

Review & Summary

Questions

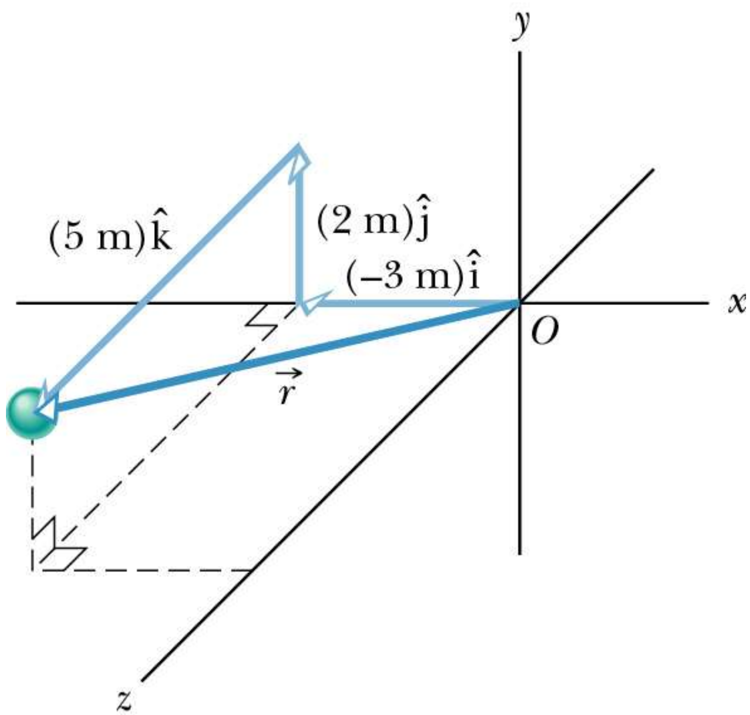
Exercises & Problems

Position & Displacement

<u>Position Vector:</u> $\vec{r} = r_x \vec{i} + r_y \vec{j} + r_z \vec{k}$

A vector from a reference point (aka the origin) to the particle.

$\vec{r} = -3\vec{i} + 2\vec{j} + 5\vec{k}$



The **vector** gives the position of the **green ball**.

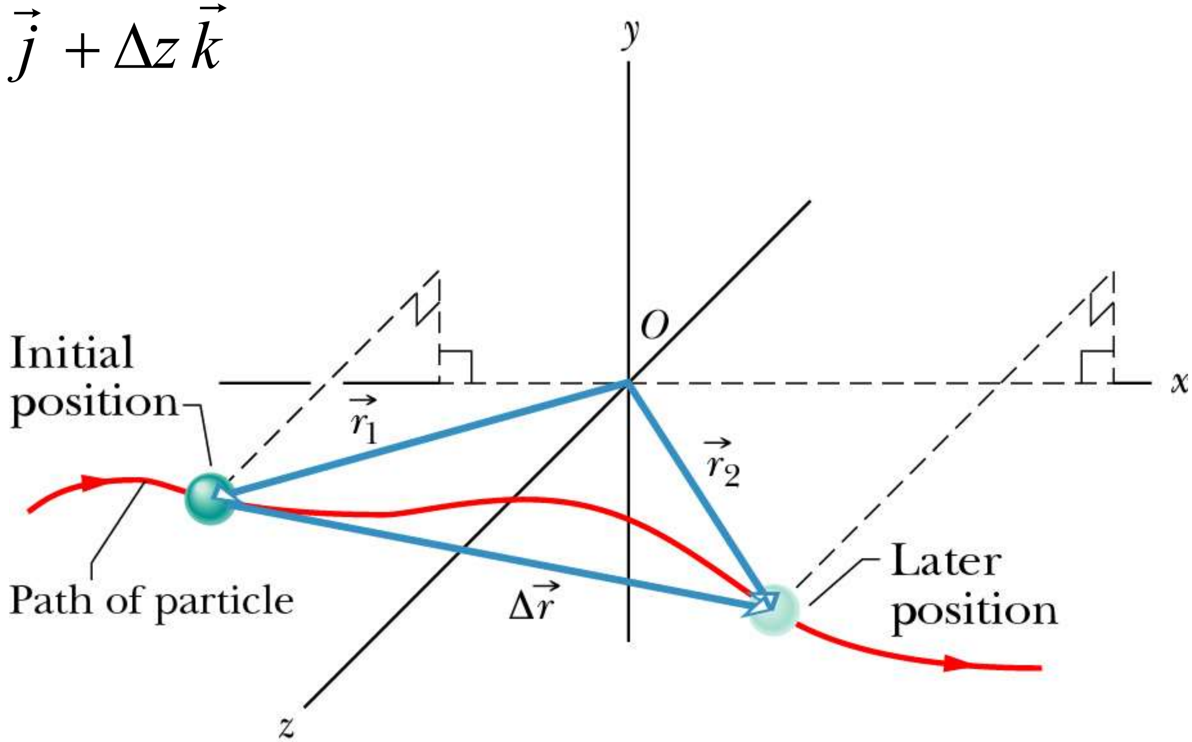
Position & Displacement

Displacement Vector: $\overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1$

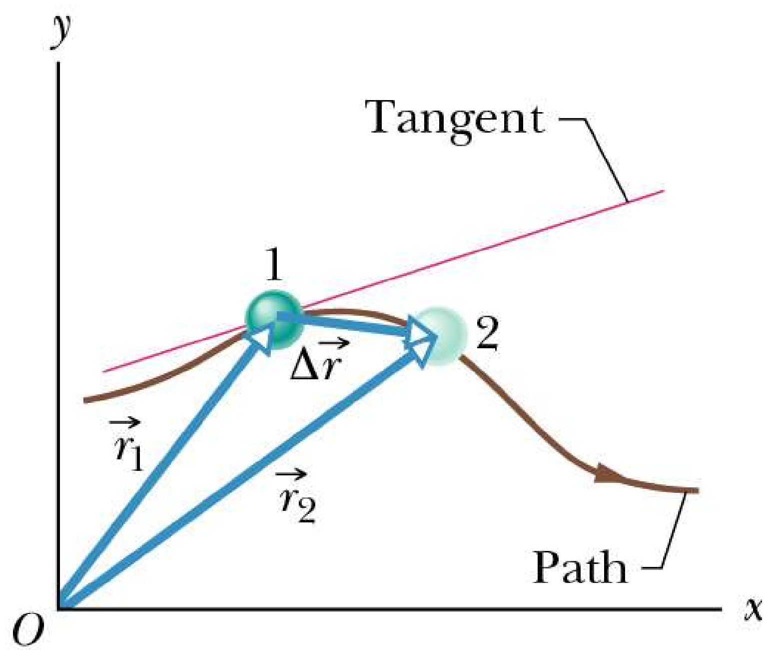
*Consider a **particle** moving along a **path**.*

$$\overrightarrow{\Delta r} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

$$\overrightarrow{\Delta r} = \Delta x\,\vec{i} + \Delta y\,\vec{j} + \Delta z\,\vec{k}$$



Average & Instantaneous Velocity



Average Velocity = ratio of the displacement to the time interval:

$$\vec{v}_{avg.} = \frac{\overrightarrow{\Delta r}}{\Delta t}$$

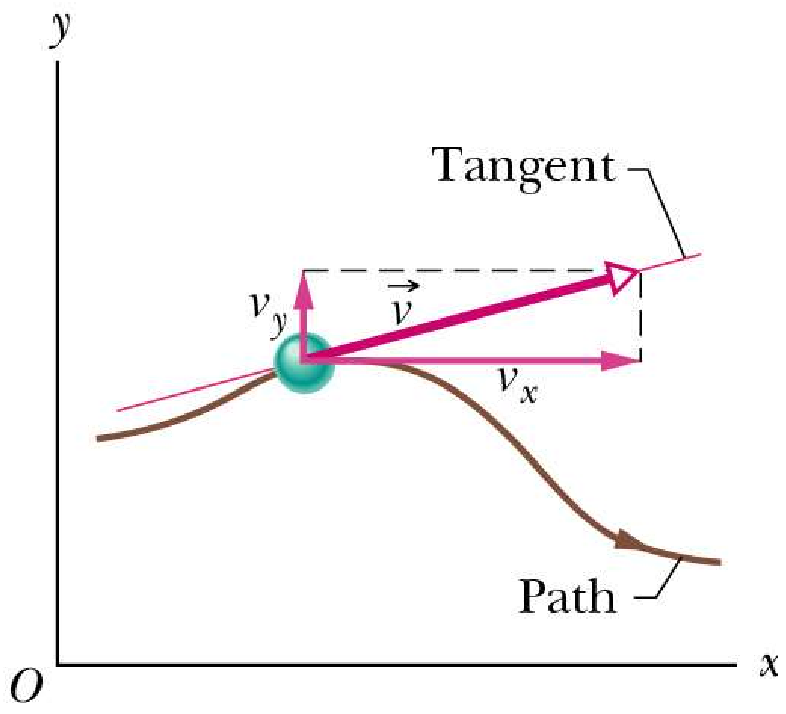
A vector in the same direction as the straight line connecting the starting point to the end point.

Instantaneous Velocity = derivative of particle position with respect to (wrt) time:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

A vector that is tangent to the particle's path at the particle's position.

Velocity Components



$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k}$$

$$v_x = \frac{dx}{dt}$$

$$v_y = \frac{dy}{dt}$$

$$v_z = \frac{dz}{dt}$$

Average & Instantaneous Acceleration

Average Acceleration = ratio of the *change in velocity* to the time interval:

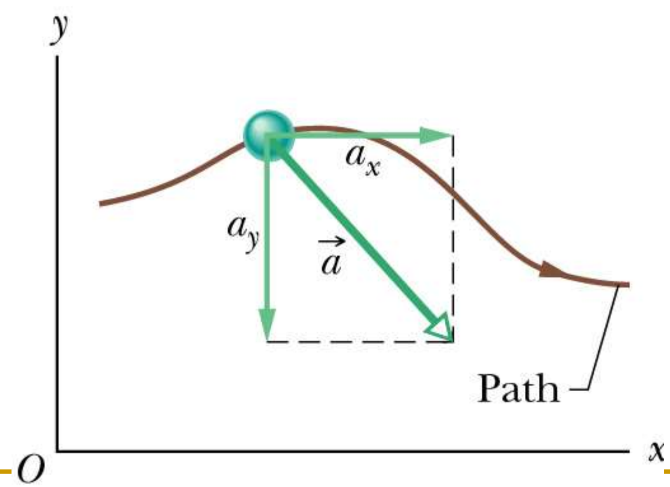
$$\vec{a}_{avg.} = \frac{\overrightarrow{\Delta v}}{\Delta t}$$

Instantaneous Acceleration = derivative of particle velocity wrt time:

$$\vec{a} = \frac{d\vec{v}}{dt} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$

If the velocity of a particle changes in either ***magnitude*** or ***direction***, the particle is subject to an acceleration.

Acceleration of a particle need not point along the path of the particle.



Projectile Motion

2-Dimensional Motion: a ***projectile*** moves in a vertical plane subject to the downward acceleration due to gravity.

The particle is launched with initial velocity $\mathbf{v}_o$:

$$\vec{v}_0 = v_{0x}\,\vec{i} + v_{0y}\,\vec{j}$$

$$v_{0x} = v_0 \cos\theta$$

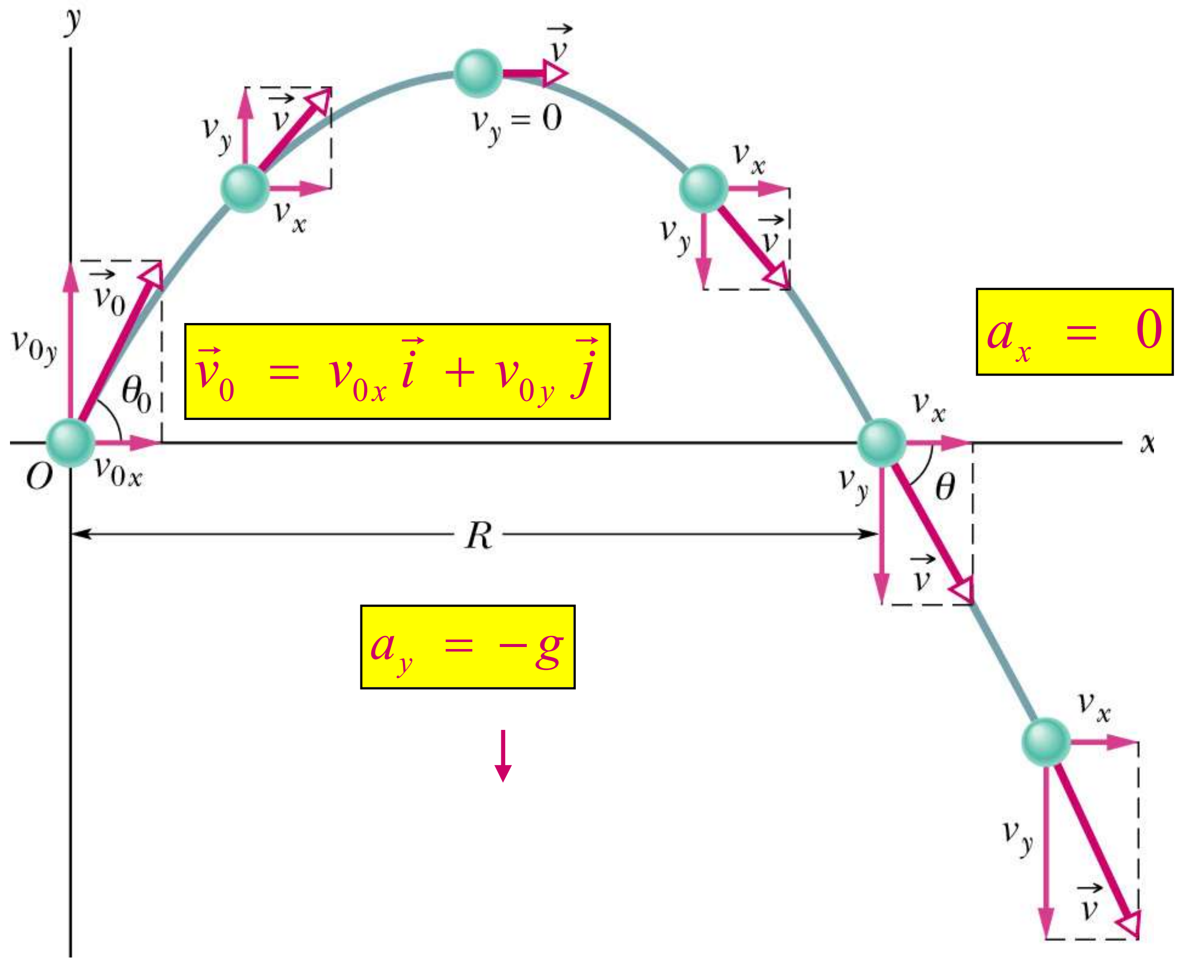
$$v_{0y} = v_0 \sin\theta$$

For a projectile, *the motions in the vertical and horizontal planes are <u>independent</u> of each other*:

horizontal motion with zero acceleration $\quad a_x = 0$

vertical motion with constant downward acceleration $\quad a_y = -g$

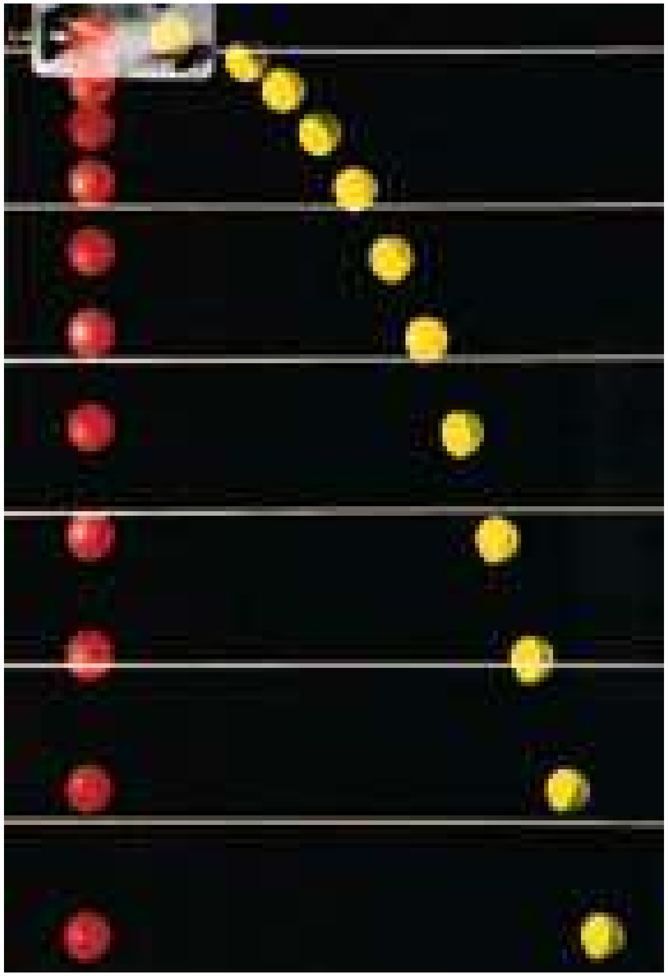
Projectile Motions



Identical Vertical Motion

Two balls are released ***simultaneously***

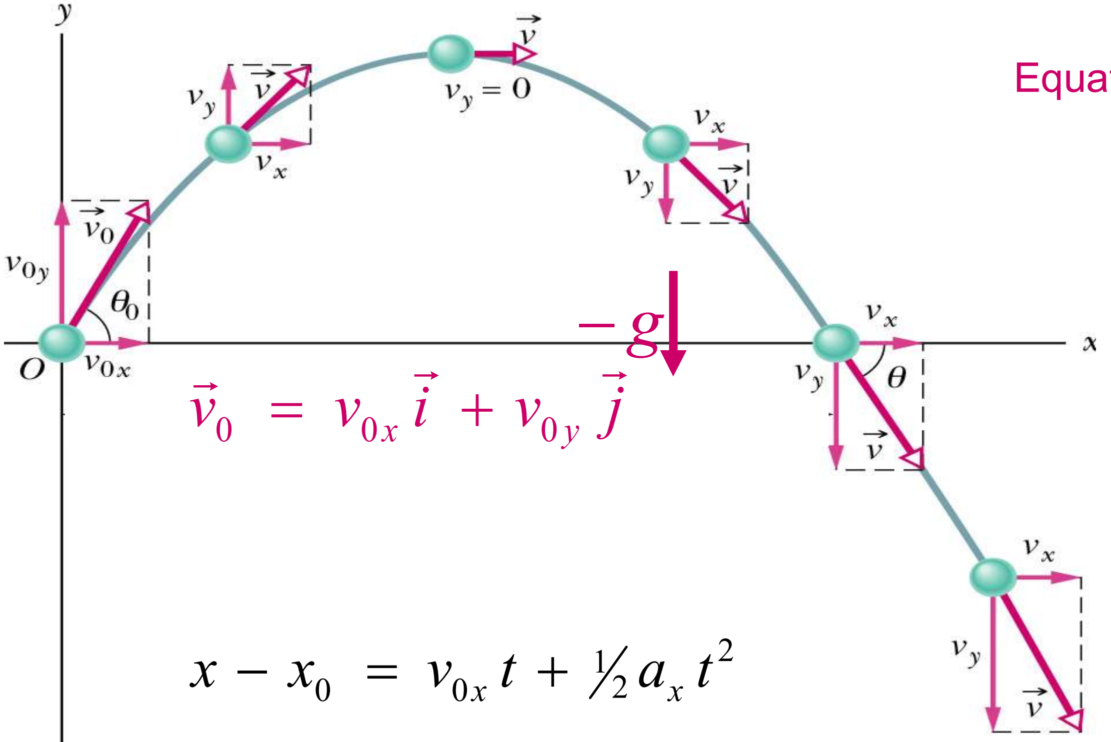
$\vec{v}_{0x} = 0$



$\vec{v}_{0x} > 0$

The vertical motions are identical.

Projectile Motions



$$\vec{v}_0 = v_{0x}\,\vec{i} + v_{0y}\,\vec{j}$$

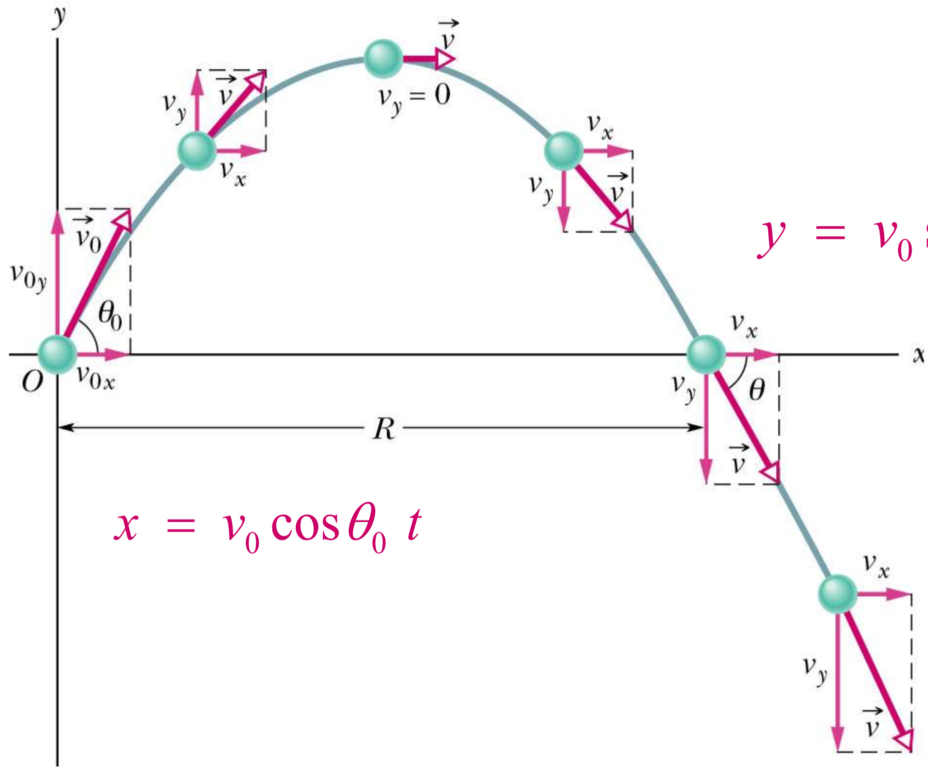
Equations that describe Projectile Motion

$$v_x = v_{0x}$$
$$v_y = -v_{0y}$$

$$x - x_0 = v_{0x}\,t + \tfrac{1}{2} a_x t^2$$
$$a_x = 0$$
$$v_x = v_{0x} = v_0 \cos\theta_0$$
$$x - x_0 = v_0 \cos\theta_0\, t$$

$$y - y_0 = v_{0y}\,t + \tfrac{1}{2} a_y t^2$$
$$a_y = -g$$
$$v_{0y} = v_0 \sin\theta_0$$
$$y - y_0 = v_0 \sin\theta_0\, t - \tfrac{1}{2} g t^2$$
$$v_y = v_0 \sin\theta - g t$$

Trajectory of a Projectile



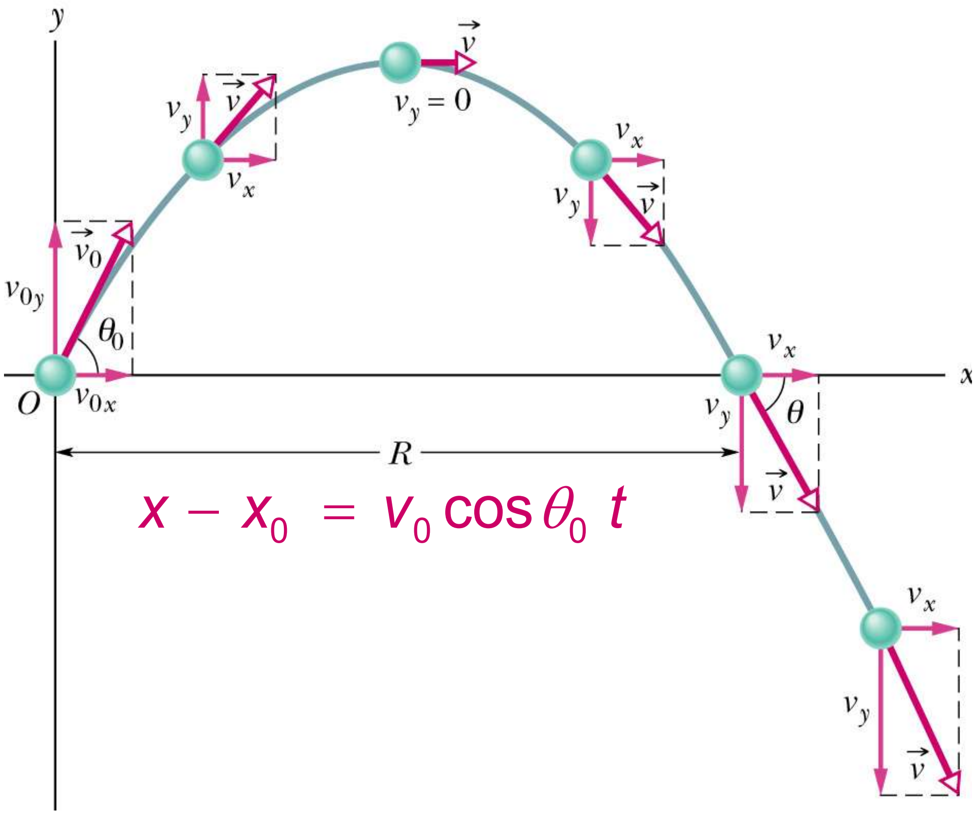
$$y = v_0 \sin\theta_0\, t - \tfrac{1}{2} g t^2$$

$$x = v_0 \cos\theta_0\, t$$

Particle Trajectory:

$$y = (\tan\theta_0)\, x - \frac{g x^2}{2(v_0 \cos\theta_0)^2}$$ *parabolic path*

Range of a Projectile



$$x - x_0 = v_0 \cos\theta_0 \, t$$

$$y - y_0 = v_0 \sin\theta_0 \, t - \tfrac{1}{2} g t^2$$

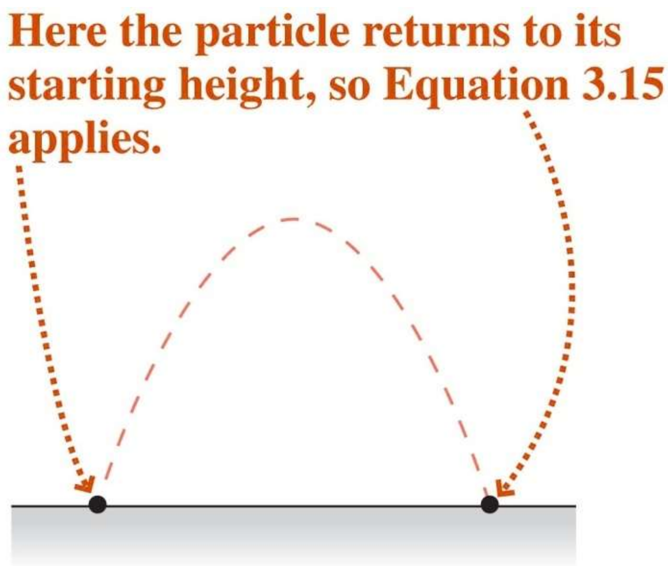
$$y - y_0 = 0$$

$$x - x_0 = \text{RANGE}$$

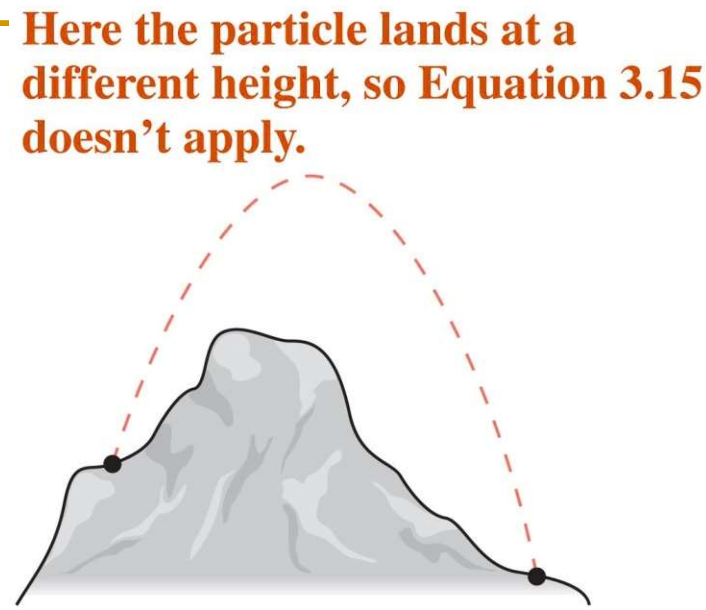
$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$R = \left(\frac{v_0^2}{g}\right) \sin 2\theta_0$$

Range equation

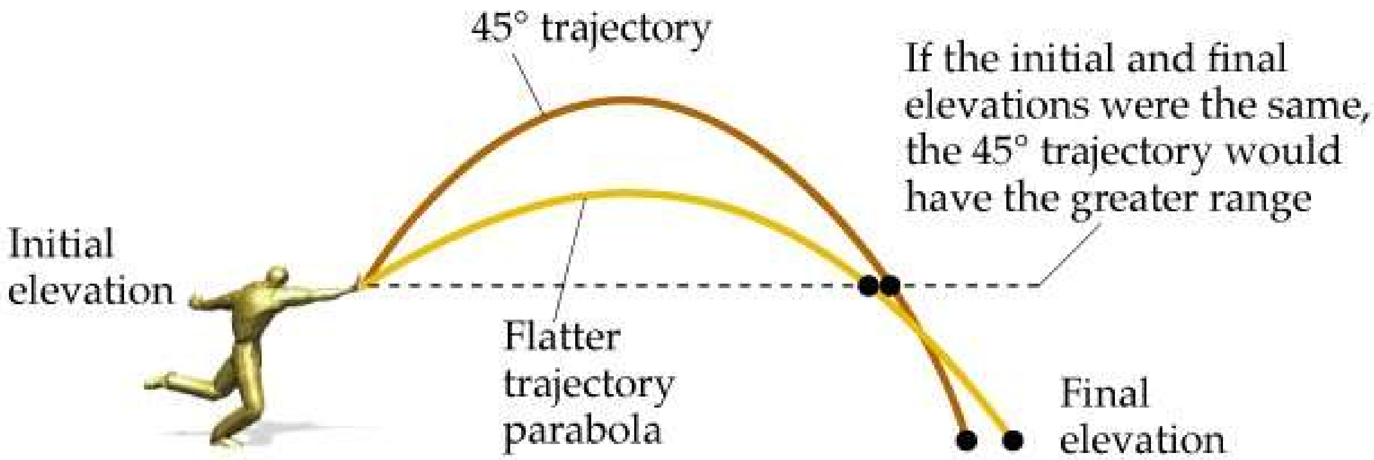


(a)



(b)

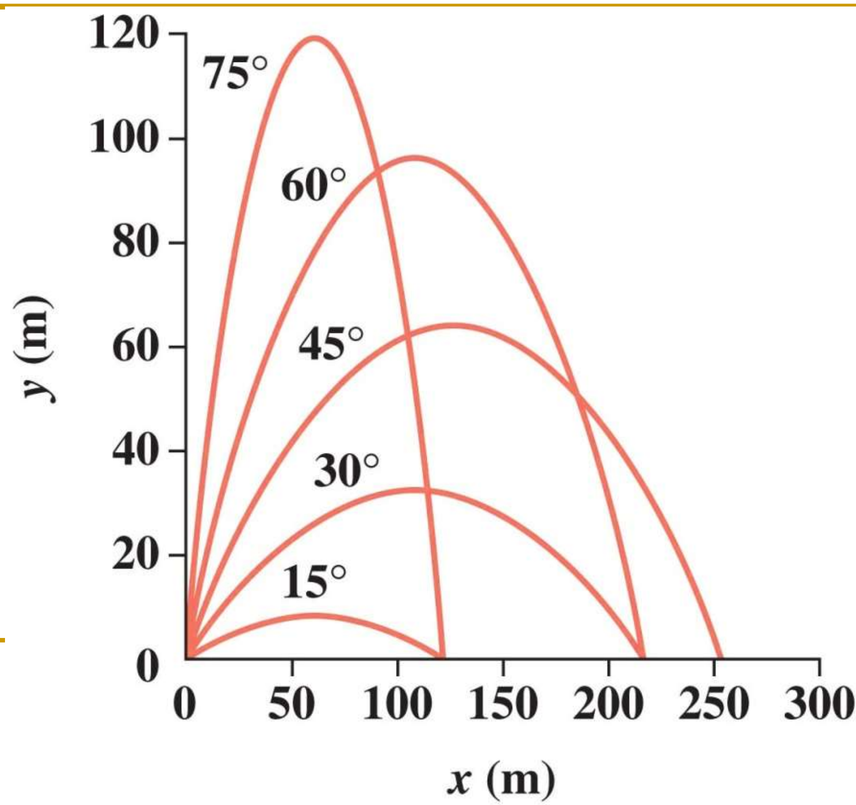
Maximum Range of a Projectile



$$R = \left(\frac{v_0^2}{g}\right)\sin 2\theta_0$$

R is a maximum for θ_0 = 45°.

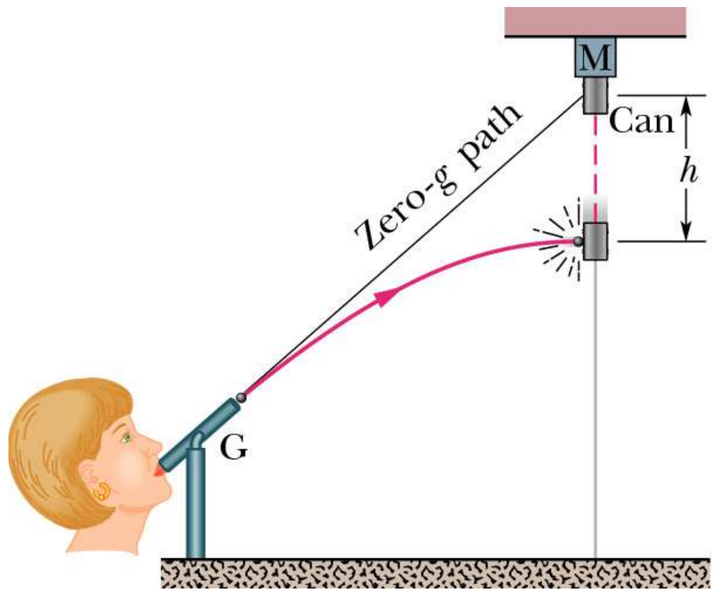
Range of a Projectile



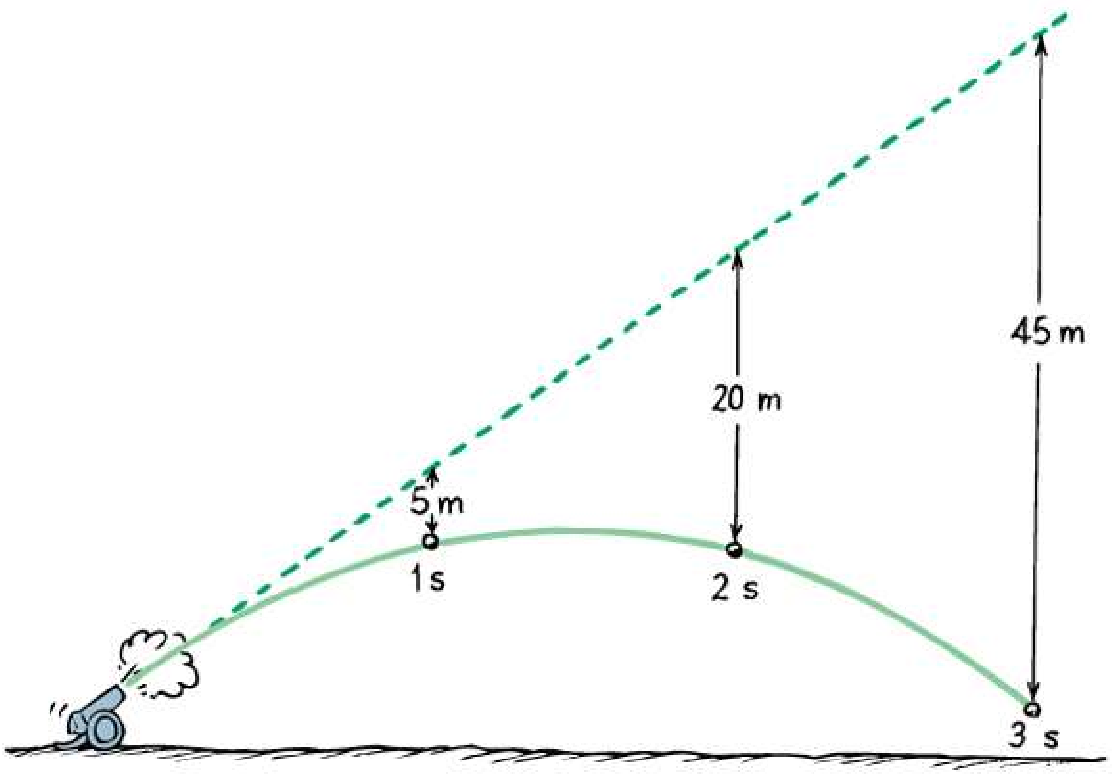
The Ball Hits the Can Every Time!

Magnet (M) releases the can just as the projectile leaves the blow gun (G).

During the time-of-flight, both the projectile and the can fall the same distance, h, under the constant acceleration –g.



Projectiles



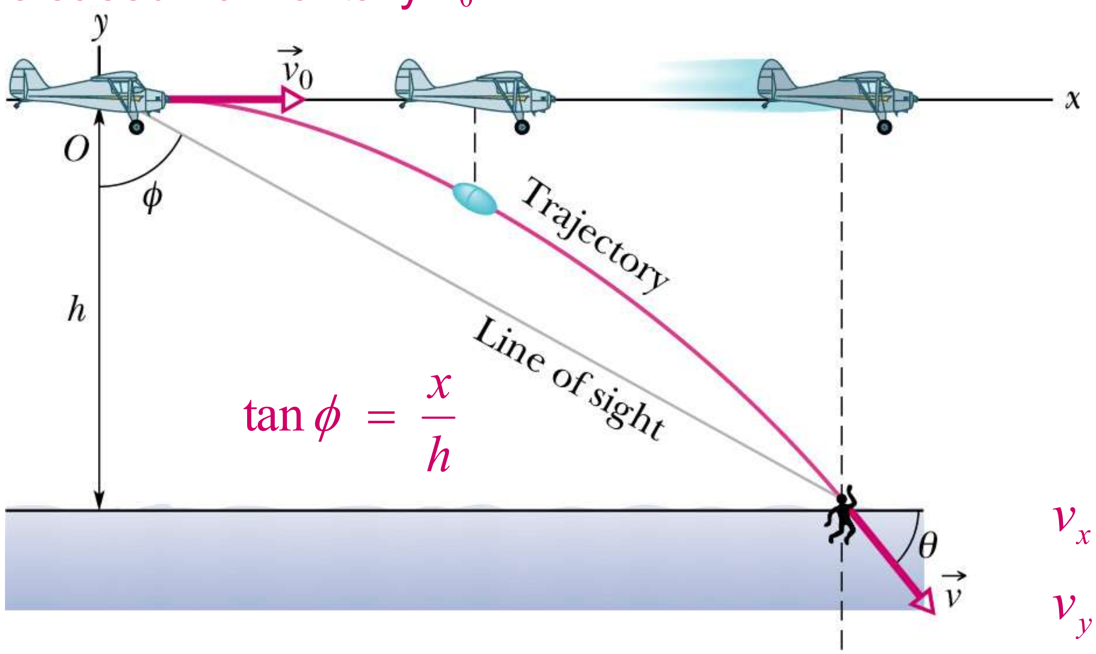
Hewitt, *Conceptual Physics*, Ninth Edition.

Object Falling from a Moving Plane

h = 500m, v_0 = 55 m/s

At what value of ϕ should the payload be released?

released horizontally $\theta_0 = 0$



$$\tan\phi = \frac{x}{h}$$

$$y = v_0 \sin\theta_0\, t - \tfrac{1}{2} g t^2$$

$$-h = -\tfrac{1}{2} g t^2$$

$$t = \sqrt{2h/g}$$

$$v_x = v_0$$

$$v_y = -g t$$

$$x = v_{0x}\, t$$

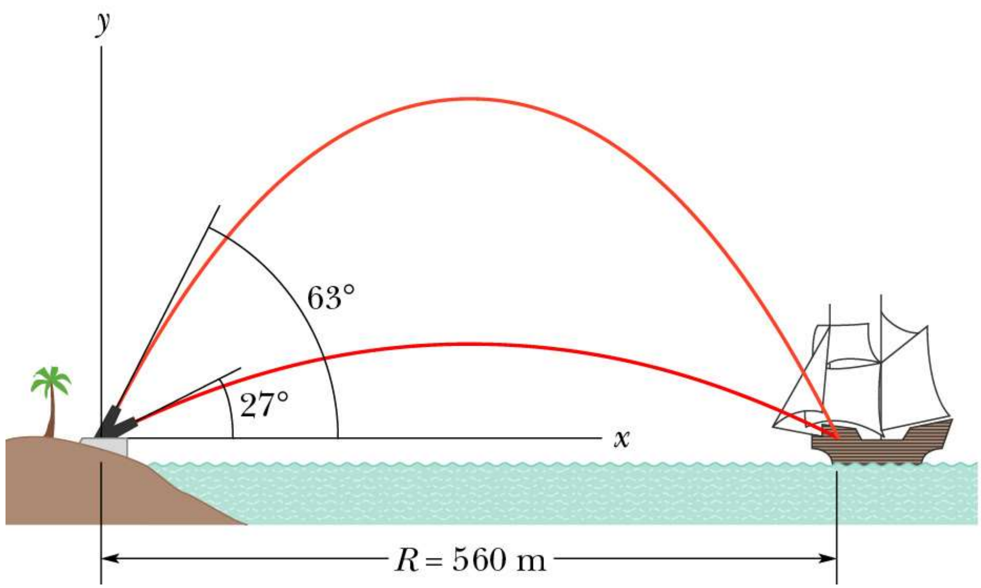
$$x = v_0 \cos\theta_0\, t$$

$$x = v_0\, t$$

Cannon Firing at Pirate Ship

V_0 cannonball = 82 m/s
Pirate ship is 560 m offshore

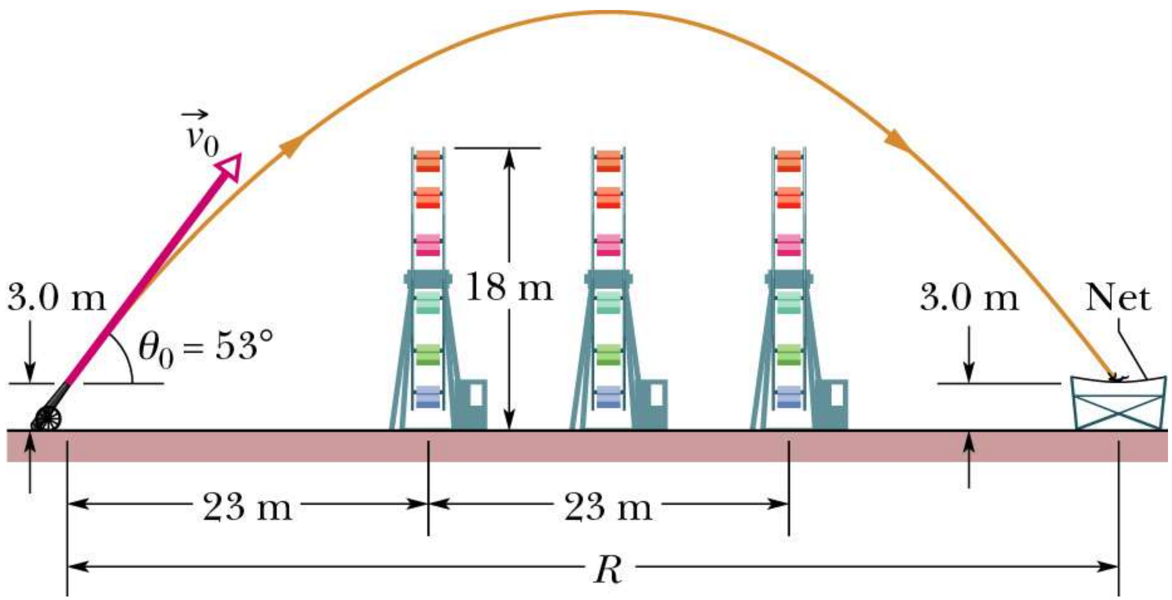
a) What θ for cannonball to hit ship
b) Safe range for the ship?



Great Zacchini

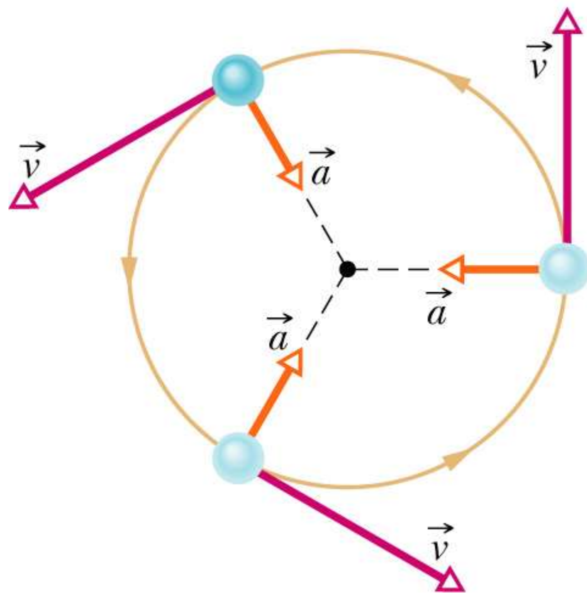
The “flight of Emanuel Zacchini” over 3 ferris wheels

v_0 = 26.5 m/s, θ_0 = 53^0, h_0 = 3.0 m
Does he clear first Ferris wheel ?
If h_{max} is above second Ferris Wheel, by how much does he clear it?



Uniform Circular Motion

The velocity *is* changing, but the speed is *not*.
Hence, the particle *is* accelerating.



$$period = T \equiv \frac{2\pi r}{v}$$

The velocity *is* always tangent to the circular path.

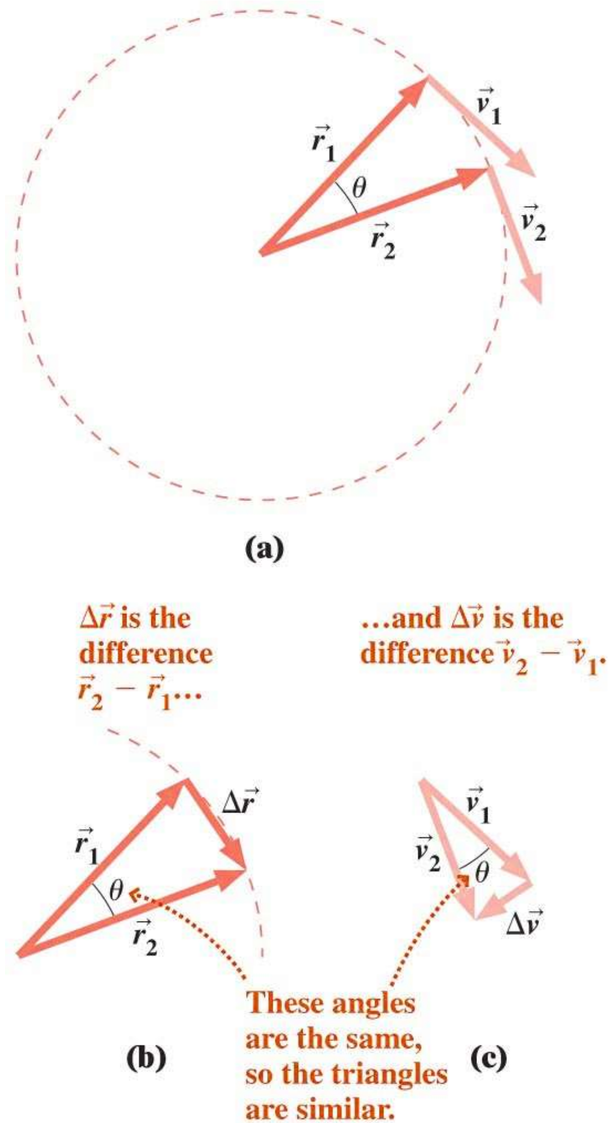
The acceleration is always directed ***radially inward***.

Centripetal Acceleration:

$$a = \frac{v^2}{r}$$

Let's prove this.

Circular Motion



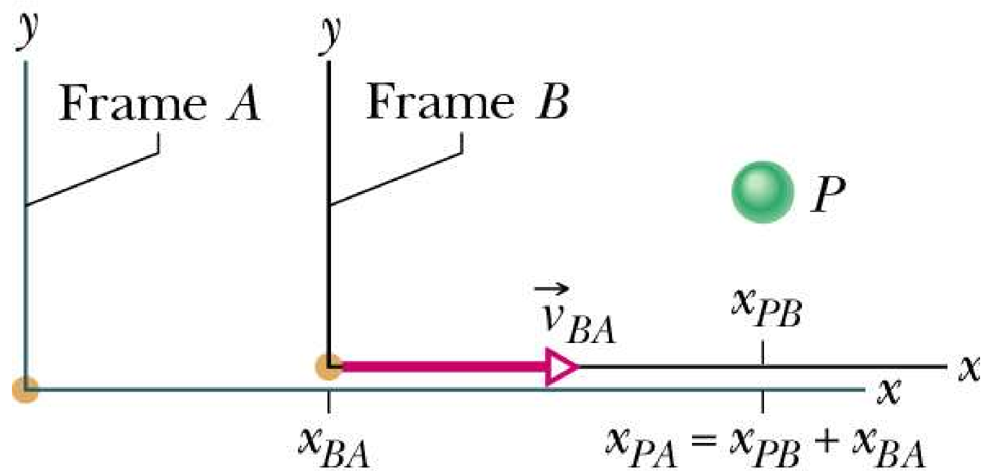
$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

$$\frac{\Delta v}{v} \cong \frac{v\Delta t}{r}$$

$$a = \frac{\Delta v}{\Delta t} \cong \frac{v^2}{r}$$

Relative Motion in One Dimension

"Reference Frame B moves @ constant velocity wrt Reference Frame A"



Differentiate wrt time

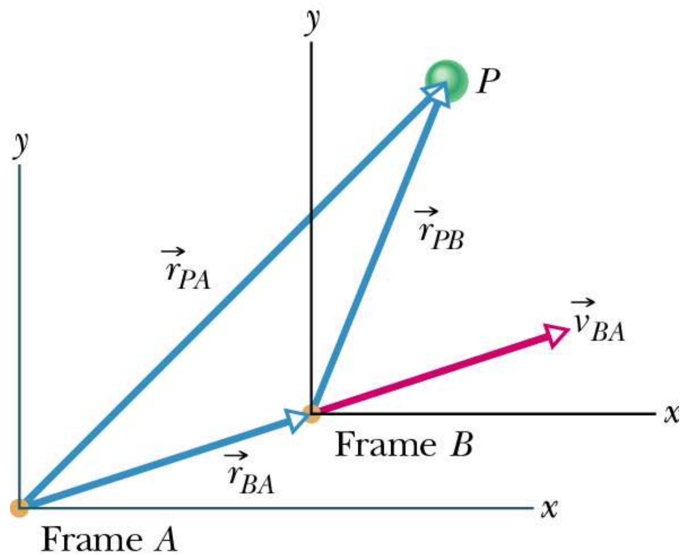
$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

Differentiate wrt time

$$\vec{a}_{PA} = \vec{a}_{PB}$$

Relative Motion in 2 Dimensions

Two *observers* watching the motion of particle P from the origins of *reference frames* A and B, while B moves at constant velocity $\mathbf{v}_{ba}$ relative to A.



$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

Differentiate wrt time

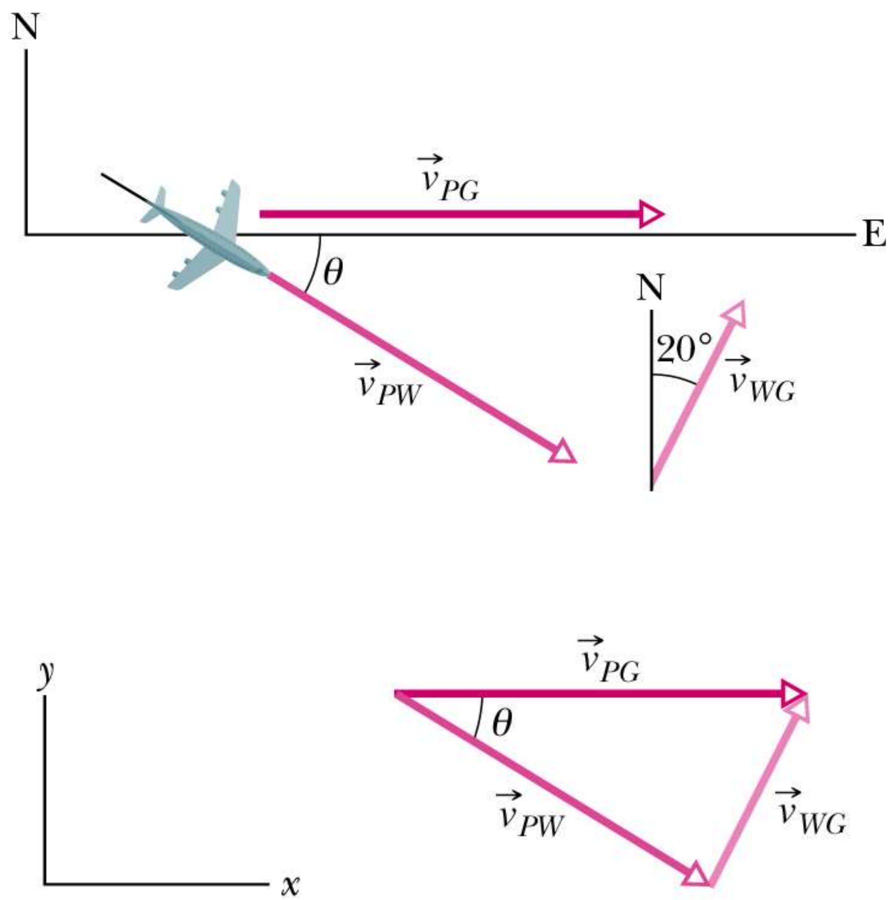
$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

Differentiate wrt time

$$\vec{a}_{PA} = \vec{a}_{PB}$$

Observers in different reference frames that move at constant velocity relative to each other will measure the *same* acceleration.

Relative Motion



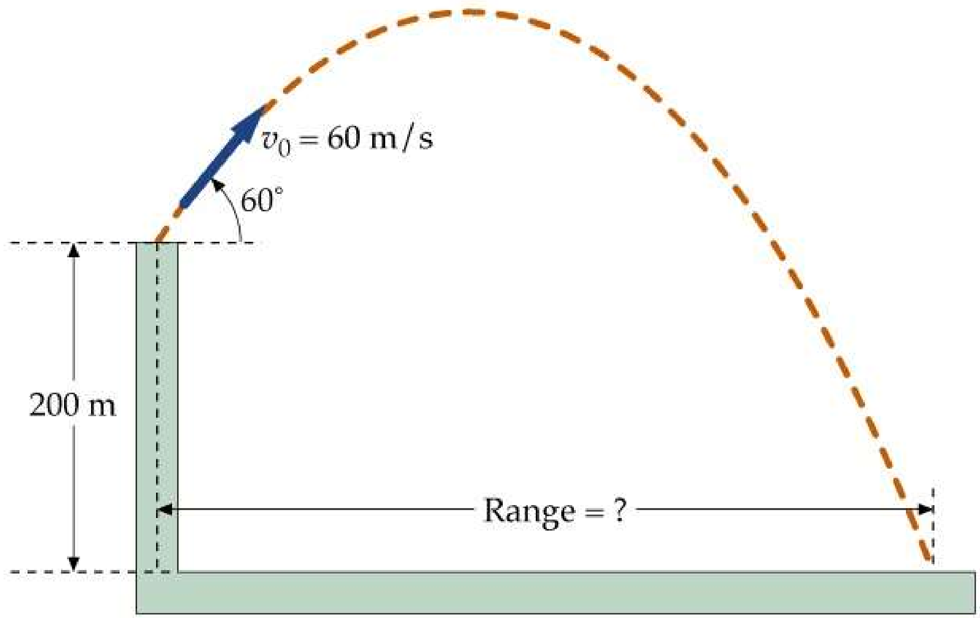
v_{pw} = 215 km/h, θ south of east

v_{wg} = 65.0 km/hr 20^0 E of N

What is v_{pg} and θ

$$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}$$

Example



$$y - y_0 = v_0 \sin\theta_0 \; t - ½ g t^2$$

$$-200m = (60 \tfrac{m}{s}) \sin 60^o t - ½ (9.8 \tfrac{m}{s^2}) t^2$$

$$t^2 - 10.60t - 40.81 = 0$$

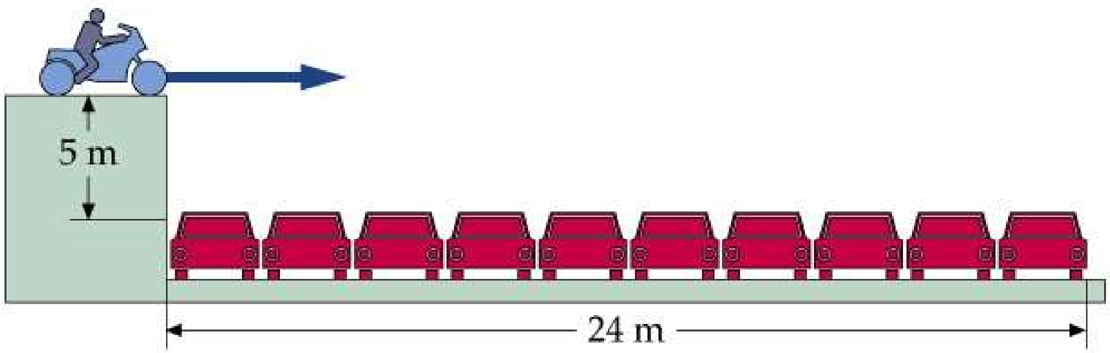
$$t = +13.6s \quad or \quad \sout{t = -3.00s}$$

$$R = x - x_0 = v_0 \cos\theta_0 \; t$$

$$R = (60 \tfrac{m}{s})(\cos 60^o)(13.6s)$$

$$R = 408m$$

Example: *How fast should she be going?*



$$y - y_0 = \tfrac{1}{2} g t^2$$

$$5m = \tfrac{1}{2}\left(9.8\, {}^{m}\!/\!_{s^2}\right) t^2$$

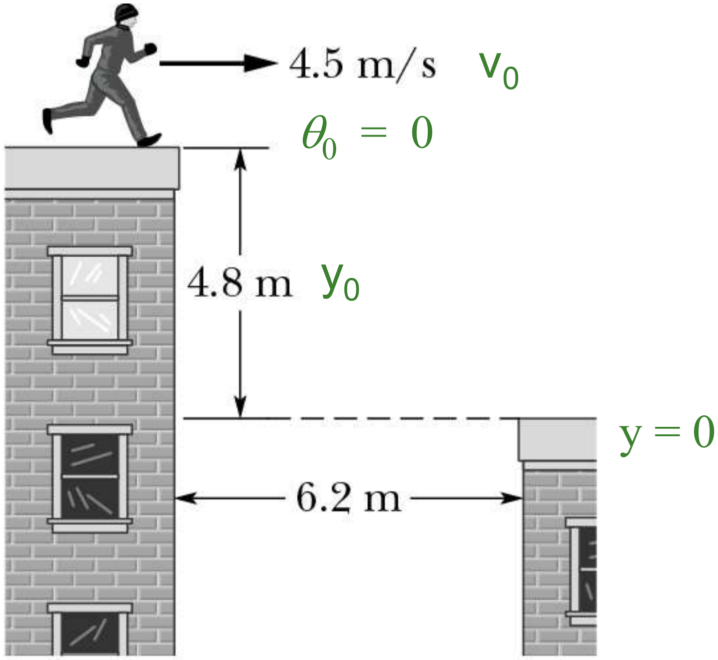
$$t = 1.01s$$

$$x - x_0 = v_0 t$$

$$v_0 = \frac{x - x_0}{t} = \frac{24m}{1.01s} = 23.8\, {}^{m}\!/\!_{s}$$

$$v_0 \approx 85\, {}^{km}\!/\!_{h} \approx 53\, {}^{mi}\!/\!_{h}$$

Example: *Can He Make the Jump?*



$$y - y_0 = v_0 \sin\theta_0 \, t - \tfrac{1}{2} g t^2$$

$t = 0.990$ s

$$x - x_0 = v_0 \cos\theta_0 \, t$$

$$x - x_0 = 4.5 \text{ m}$$

He can't make it!